Section 1: Introduction to complex numbers

Solutions to Exercise level 2

1. $w = \frac{5+2i}{z} = \frac{5+2i}{-3+4i}$

$= \frac{(5+2i)(-3-4i)}{(-3+4i)(-3-4i)}$

$= \frac{-15-6i-20i+8}{9+16}$

$= \frac{-7-26i}{25}$

2. $z = (a+i)^4$

$= a^4 + 4a^3i + 6a^2i^2 + 4ai^3 + i^4$

$= a^4 + 4a^3i - 6a^2 - 4ai + 1$

(i) If z is real, $4a^3 - 4a = 0$

$4a(a^2 - 1) = 0$

$4a(a+1)(a-1) = 0$

so $a = 0$, -1 or 1.

(ii) If z is wholly imaginary, $a^4 - 6a^2 + 1 = 0$

$a^2 = \frac{6 \pm \sqrt{36-4}}{2} = 3 \pm 2\sqrt{2}$

$a = \pm\sqrt{3 \pm 2\sqrt{2}}$

3. $(a+bi)^* = (a+bi)^2$

$a - bi = a^2 + 2abi - b^2$

Equating imaginary parts: $-b = 2ab$

$b + 2ab = 0$

$b(1+2a) = 0$

$b = 0$ or $a = -\frac{1}{2}$

Equating real parts: $a = a^2 - b^2$

If $b = 0$: $a = a^2$

$a(1-a) = 0$

$a = 0$ or 1

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If $a = -\frac{1}{2}: -\frac{1}{2} = \frac{1}{4} - b^2$

$b^2 = \frac{3}{4}$

$b = \pm\frac{1}{2}\sqrt{3}$

The possible values of a and b are: $a = b = 0$

$a = 1,\ b = 0$

$a = -\frac{1}{2},\ b = \pm\frac{1}{2}\sqrt{3}$

4. (i) $\dfrac{1}{3+2i} + \dfrac{1}{3-2i} = \dfrac{3-2i+3+2i}{(3+2i)(3-2i)}$

$= \dfrac{6}{9+4}$

$= \dfrac{6}{13}$

(ii) $3 + i + \dfrac{4}{3-i} = 3 + i + \dfrac{4(3+i)}{(3-i)(3+i)}$

$= 3 + i + \dfrac{4(3+i)}{9+1}$

$= 3 + i + \frac{2}{5}(3+i)$

$= \frac{7}{5}(3+i)$

(iii) $\dfrac{3}{1-i} - \dfrac{2i}{2+i} = \dfrac{3(1+i)}{(1-i)(1+i)} - \dfrac{2i(2-i)}{(2+i)(2-i)}$

$= \dfrac{3+3i}{2} - \dfrac{4i+2}{5}$

$= \dfrac{15+15i-8i-4}{10}$

$= \dfrac{11+7i}{10}$

5. (i) $(a+bi)(2+i) = a - 3i$

$2a + ai + 2bi - b = a - 3i$

Equating real parts: $2a - b = a$

$a = b$

Equating imaginary parts: $a + 2b = -3$

$3a = -3$

$a = -1$

$a = -1,\ b = -1$

(ii) $(a+i)(4-bi)=3b+2ai$

$4a-abi+4i+b=3b+2ai$

Equating real parts: $4a+b=3b$

$2a=b$

$-ab+4=2a$

$-2a^2+4=2a$

Equating imaginary parts: $a^2+a-2=0$

$(a+2)(a-1)=0$

$a=-2$ or $a=1$

$a = -2$, $b = -4$ or $a = 1$, $b = 2$.

6. $(a+bi)^2=3-4i$

$a^2+2abi-b^2=3-4i$

Equating imaginary parts: $2ab=-4$

$b=-\frac{2}{a}$

Equating real parts: $a^2-b^2=3$

$a^2-\frac{4}{a^2}=3$

$a^4-4=3a^2$

$a^4-3a^2-4=0$

$(a^2-4)(a^2+1)=0$

Since a is real, $a=\pm 2$ so $b=\mp 1$

The square roots of $3-4i$ are $2-i$ and $-2+i$.

7. (i) One root is $2-i$ so the other root is $2+i$

Equation is $(z-2+i)(z-2-i)=0$

$(z-2)^2+1=0$

$z^2-4z+4+1=0$

$z^2-4z+5=0$

$p = -4$, $q = 5$

(ii) One root is $1-3i$ so the other root is $1+3i$

Equation is $(z-1+3i)(z-1-3i)=0$

$(z-1)^2+9=0$

$z^2-2z+1+9=0$

$z^2-2z+10=0$

$p = -2$, $q = 10$

(iii) One root is 2i so the other root is -2i

Equation is $(z-2i)(z+2i)=0$

$$z^2+4=0$$

$p=0,\ q=4$

(iv) One root is 5 – 3i so the other root is 5 + 3i

Equation is $(z-5+3i)(z-5-3i)=0$

$$(z-5)^2+9=0$$

$$z^2-10z+25+9=0$$

$$z^2-10z+34=0$$

$p=-10,\ q=34$

8. $\dfrac{5}{a+bi}+\dfrac{2}{1+3i}=1$

$$\frac{5}{a+bi}=1-\frac{2}{1+3i}=\frac{1+3i-2}{1+3i}=\frac{-1+3i}{1+3i}$$

$$\frac{a+bi}{5}=\frac{1+3i}{-1+3i}$$

$$=\frac{(1+3i)(-1-3i)}{(-1+3i)(-1-3i)}$$

$$=\frac{-1-6i+9}{1+9}$$

$$=\frac{8-6i}{10}$$

$$=\frac{4-3i}{5}$$

$a+bi=4-3i$