Section 1: Introduction to complex numbers

Solutions to Exercise level 1

1. (i) $z^2 + 25 = 0$

$z^2 = -25$

$z = \pm 5i$

(ii) $4z^2 + 9 = 0$

$z^2 = -\frac{9}{4}$

$z = \pm \frac{3}{2}i$

(iii) $z^2 - 2z + 2 = 0$

$$z = \frac{2 \pm \sqrt{4 - 4 \times 1 \times 2}}{2}$$

$$= \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{2 \pm 2i}{2}$$

$= 1 \pm i$

(iv) $4z^2 + 4z + 5 = 0$

$$z = \frac{-4 \pm \sqrt{16 - 4 \times 4 \times 5}}{8}$$

$$= \frac{-4 \pm \sqrt{-64}}{8}$$

$$= \frac{-4 \pm 8i}{8}$$

$= -\frac{1}{2} \pm i$

2. (i) $2z - 3w = 2(4 - 3i) - 3(2 + i)$

$= 8 - 6i - 6 - 3i$

$= 2 - 9i$

(ii) $zw = (4 - 3i)(2 + i)$

$= 8 - 6i + 4i - 3i^2$

$= 8 - 2i + 3$

$= 11 - 2i$

(iii) $iz = i(4-3i) = 4i - 3i^2 = 4i + 3$

$$(iz)^2 = (4i+3)^2$$
$$= 16i^2 + 24i + 9$$
$$= -16 + 24i + 9$$
$$= -7 + 24i$$

(iv) $z * w = (4-3i) * (2+i)$

$$= (4+3i)(2+i)$$
$$= 8 + 6i + 4i + 3i^2$$
$$= 8 + 10i - 3$$
$$= 5 + 10i$$

3. (i) (a) $z_1 + z_2 = 2 + 3i + 1 - 2i = 3 + i$

(b) $z_1 - z_2 = 2 + 3i - 1 + 2i = 1 + 5i$

(c) $z_1 z_2 = (2+3i)(1-2i) = 2 - 4i + 3i + 6 = 8 - i$

(d) $z_1^* = 2 - 3i$

(e) $z_2^* = 1 + 2i$

(f) $z_1^* + z_2^* = 2 - 3i + 1 + 2i = 3 - i$

(g) $z_1^* - z_2^* = 2 - 3i - 1 - 2i = 1 - 5i$

(h) $z_1^* z_2^* = (2-3i)(1+2i) = 2 + 4i - 3i + 6 = 8 + i$

(ii) (a) $z_1 + z_2 = -2i + 3 + i = 3 - i$

(b) $z_1 - z_2 = -2i - 3 - i = -3 - 3i$

(c) $z_1 z_2 = -2i(3+i) = -6i + 2 = 2 - 6i$

(d) $z_1^* = 2i$

(e) $z_2^* = 3 - i$

(f) $z_1^* + z_2^* = 2i + 3 - i = 3 + i$

(g) $z_1^* - z_2^* = 2i - (3 - i) = -3 + 3i$

(h) $z_1^* z_2^* = 2i(3-i) = 6i + 2 = 2 + 6i$

$z_1^* + z_2^* = (z_1 + z_2)^*$

$z_1^* - z_2^* = (z_1 - z_2)^*$

$z_1^* z_2^* = (z_1 z_2)^*$

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4. $z = 2 \pm 3i$

$(z-2) = \pm 3i$

$(z-2)^2 = -9$

$z^2 - 4z + 4 = -9$

$z^2 - 4z + 13 = 0$

5. (i) $\dfrac{2}{3+i} = \dfrac{2(3-i)}{(3+i)(3-i)}$

$= \dfrac{2(3-i)}{9+1}$

$= \dfrac{2(3-i)}{10} = \dfrac{3-i}{5}$

(ii) $\dfrac{2-i}{1+2i} = \dfrac{(2-i)(1-2i)}{(1+2i)(1-2i)}$

$= \dfrac{2-i-4i-2}{1+4}$

$= \dfrac{-5i}{5} = -i$

6. $(2+i)z = 3+4i$

$z = \dfrac{3+4i}{(2+i)}$

$= \dfrac{(3+4i)(2-i)}{(2+i)(2-i)}$

$= \dfrac{6-3i+8i+4}{4+1}$

$= \dfrac{10+5i}{5} = 2+i$

7. (i) The other root is $1-2i$

(ii) $z = 1 \pm 2i$

$(z-1) = \pm 2i$

$(z-1)^2 = -4$

$z^2 - 2z + 1 = -4$

$z^2 - 2z + 5 = 0$

So $a = -2, b = 5$