Section 1: Solving problems

Solutions to Exercise level 3

1. Let the number of red balls initially be n

Proportion of red balls at first is $\frac{n}{140}$

When 30 red balls are added, proportion of red balls is $\frac{n+30}{170}$

Proportion has doubled, so $\frac{n+30}{170} = 2 \times \frac{n}{140}$

$$\frac{n+30}{170} = \frac{2n}{140}$$

$$140(n+30) = 2n \times 170$$

$$140n + 4200 = 340n$$

$$200n = 4200$$

$$n = 21$$

There were 21 red balls to begin with.

2. Suppose 45 can be written as the sum of 2 consecutive integers.

Then $n + (n+1) = 45$

$$2n + 1 = 45$$

$$n = 22$$

so $22 + 23 = 45$

Suppose 45 can be written as the sum of 3 consecutive integers

Then $(2n+1) + (n+2) = 45$

$$3n + 3 = 45$$

$$n = 14$$

so $14 + 15 + 16 = 45$

From above the sum of two consecutive integers is 2n+1, then add the next integer, n+2

Suppose 45 can be written as the sum of 4 consecutive integers

Then $(3n+3) + (n+3) = 45$

$$4n + 6 = 45$$

This does not have an integer solution

Suppose 45 can be written as the sum of 5 consecutive integers

Then $(4n+6) + (n+4) = 45$

$$5n + 10 = 45$$

$$n = 7$$

so $7 + 8 + 9 + 10 + 11 = 45$

Suppose 45 can be written as the sum of 6 consecutive integers
Then $(5n+10)+(n+5)=45$

$6n+15=45$

$n=5$

so $5+6+7+8+9+10=45$

Suppose 45 can be written as the sum of 7 consecutive integers
Then $(6n+15)+(n+6)=45$

$7n+21=45$

This does not have an integer solution

Suppose 45 can be written as the sum of 8 consecutive integers
Then $(7n+21)+(n+7)=45$

$8n+28=45$

This does not have an integer solution

Suppose 45 can be written as the sum of 9 consecutive integers
Then $(8n+28)+(n+8)=45$

$9n+36=45$

$n=1$

so $1+2+3+4+5+6+7+8+9=45$

So there are 5 different ways of writing 45 as the sum of consecutive integers.

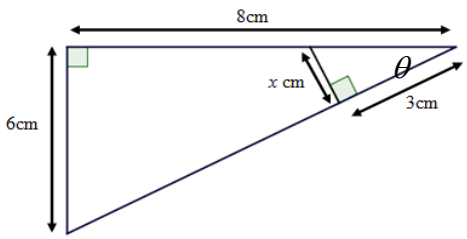
3. From small triangle, $\tan\theta=\frac{x}{3}$

From large triangle, $\tan\theta=\frac{6}{8}$

so $\frac{x}{3}=\frac{6}{8}$

$8x=18$

$x=2.25$



4. Suppose n is a positive integer which leaves a remainder of 11 when divided into 1000.
Then $kn+11=1000$ where k is an integer
so $kn=989$
The only factors of 989 are 1, 23, 43 and 989
so 989 can be written as 1×989 or 23×43
n cannot be 1 as $1000\div1=1000$ with no remainder
n can be 23, 43 or 989

so there are three possible positive integers.