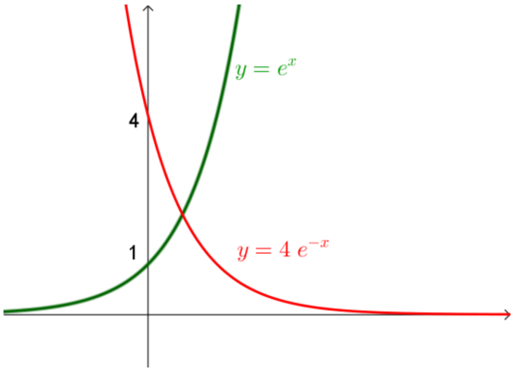
Section 2: Natural logarithms and exponentials

Exercise level 3 solutions

1. (i)

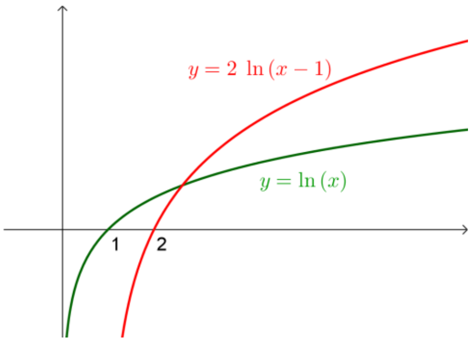


(ii) At intersection point, $e^x = 4e^{-x}$

$$e^{2x} = 4$$
$$2x = \ln 4 = 2\ln 2$$
$$x = \ln 2$$

When $x = \ln 2$, $y = e^{\ln 2} = 2$, so intersection point is $(\ln 2, 2)$.

(iii)



At intersection point, $\ln x = 2\ln(x-1)$

$$\ln x = \ln(x-1)^2$$
$$x = (x-1)^2$$
$$x = x^2 - 2x + 1$$
$$x^2 - 3x + 1 = 0$$
$$x = \frac{3 \pm \sqrt{3^2 - 4\times 1\times 1}}{2} = \frac{3\pm\sqrt{5}}{2}$$

$x = \dfrac{3-\sqrt{5}}{2}$ is not a possible solution as this would mean that $x-1$ is negative.

So $x = \frac{3+\sqrt{5}}{2}$

When $x = \frac{3+\sqrt{5}}{2}$, $y = \ln\left(\frac{3+\sqrt{5}}{2}\right)$

so intersection point is $\left(\frac{3+\sqrt{5}}{2}, \ln\left(\frac{3+\sqrt{5}}{2}\right)\right)$

2. (i) $10^x = \left(e^{\ln 10}\right)^x = e^{x\ln 10}$

(ii) Let $y = \ln x$

Then $x = e^y$

$$\log x = \log e^y$$
$$\log x = y\log e$$
$$y = \frac{\log x}{\log e}$$

so $\ln x = \frac{\log x}{\log e}$

3. (i) $\ln(\ln x^e) - \ln(\ln x) = \ln(e\ln x) - \ln(\ln x)$

$$= \ln e + \ln(\ln x) - \ln(\ln x)$$
$$= \ln e$$
$$= 1$$

(ii) $e^{\ln x} + \ln e^x = 3$

$$x + x\ln e = 3$$
$$x + x = 3$$
$$x = \tfrac{3}{2}$$

(iii) $e^{2\ln x} + 2\ln e^x = 3$

$$(e^{\ln x})^2 + 2x\ln e = 3$$
$$x^2 + 2x = 3$$
$$x^2 + 2x - 3 = 0$$
$$(x+3)(x-1) = 0$$
$$x = -3 \text{ or } 1$$

x cannot be negative, so $x = 1$

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4. (i) $f(x) = 2x^3 - x^2 - 8x + 4$

$f(\frac{1}{2}) = 2(\frac{1}{2})^3 - (\frac{1}{2})^2 - 8 \times \frac{1}{2} + 4$

$= \frac{1}{4} - \frac{1}{4} - 4 + 4$

$= 0$

$x = \frac{1}{2}$ is a root so $(2x-1)$ is a factor.

(ii) Let $y = e^x$

$2y^3 - y^2 - 8y + 4 = 0$

$(2y-1)(y^2-4) = 0$

$(2y-1)(y-2)(y+2) = 0$

$y = \frac{1}{2}, 2, -2$

so $e^x = \frac{1}{2} \quad \Rightarrow x = \ln\frac{1}{2} = -\ln 2$

or $e^x = 2 \quad \Rightarrow x = \ln 2$

or $e^x = -2$ - no real solution

so $x = \pm\ln 2$