Section 2: Natural logarithms and exponentials

Solutions to Exercise level 2

1. $a = be^{-kx}$ $\qquad\qquad y = e^x$

$$\frac{a}{b} = e^{-kx}$$

$$\ln\left(\frac{a}{b}\right) = -kx$$

$$x = -\frac{1}{k}\ln\left(\frac{a}{b}\right) = \frac{1}{k}\ln\left(\frac{b}{a}\right)$$

2. $\ln x = a$

$$x = e^a$$

3. $\ln y - \ln x = t$

$$\ln\left(\frac{y}{x}\right) = t$$

$$\frac{y}{x} = e^t$$

$$y = xe^t$$

$$x = ye^{-t}$$

4. (i) $m = m_0 e^{-kt}$

When $t = 120$, $m = \frac{1}{2}m_0$.

$$\tfrac{1}{2}m_0 = m_0 e^{-120k}$$

$$e^{-120k} = \tfrac{1}{2}$$

$$-120k = \ln\tfrac{1}{2}$$

$$k = 0.00578$$

(ii) $m = m_0 e^{-0.00578t}$

When $m = 0.05m_0$, $0.05m_0 = m_0 e^{-0.00578t}$

$$e^{-0.00578t} = 0.05$$

$$-0.00578t = \ln 0.05$$

$t = 520$ seconds (to nearest 10 seconds)

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5. (i) $N = 50e^{0.1t}$

When N = 200

$200 = 50e^{0.1t}$

$e^{0.1t} = 4$

$0.1t = \ln 4$

$t = 13.86$

The population is greater than 200 after 14 weeks.

(ii) $\frac{dN}{dt} = 50 \times 0.1e^{0.1t} = 5e^{0.1t}$

When t = 5, $\frac{dN}{dt} = 5e^{0.5} = 8.24$

Rate of increase after 5 weeks is 8.24 mice / week.

(iii) $N = 50e^{0.1t} \quad \Rightarrow e^{0.1t} = \frac{N}{50}$

$\frac{dN}{dt} = 5 \times \frac{N}{50}$

$\frac{dN}{dt} = \frac{N}{10}$

The value of k is 0.1.

(iv) When N = 200, $\frac{dN}{dt} = \frac{200}{10} = 20$

The rate of increase is 20 mice / week.

(v) Resources such as food and space are unlikely to be able to sustain the population as it becomes very large. Also, the model takes no account of mice dying.