Section 2: Natural logarithms and exponentials

Solutions to Exercise level 1

1. (i) $e^2 = 7.389$ (4 s.f.)
   (ii) $e^{-3} = 0.04979$ (4 s.f.)
   (iii) $e^{-0.6} = 0.5488$ (4 s.f.)
   (iv) $\ln 2 = 0.6931$ (4 s.f.)
   (v) $\ln 0.3 = -1.204$ (4 s.f.)
   (vi) $\ln 5 = 1.609$ (4 s.f.)

2. (i) $e^x = 2$
   $x = \ln 2 = 0.693$ (3 s.f.)

   (ii) $e^{2x-1} = 3$
   $2x - 1 = \ln 3$
   $2x = \ln 3 + 1$
   $x = \frac{1}{2}(\ln 3 + 1) = 1.05$ (3 s.f.)

   (iii) $e^x = 2e^{1-2x}$
   $x = \ln\left(2e^{1-2x}\right) = \ln 2 + \ln e^{1-2x} = \ln 2 + 1 - 2x$
   $3x = \ln 2 + 1$
   $x = \frac{1}{3}(\ln 2 + 1) = 0.564$ (3 s.f.)

   (iv) $\ln x = 5$
   $x = e^5 = 148$ (3 s.f.)

   (v) $\ln x^2 = -2$
   $2\ln x = -2$
   $\ln x = -1$
   $x = e^{-1} = 0.368$ (3 s.f.)

   (vi) $\ln x = 3 - \ln 2x$
   $\ln x + \ln 2x = 3$
   $\ln 2x^2 = 3$
   $2x^2 = e^3$
   $x = \sqrt{\frac{1}{2}e^3} = 3.17$ (3 s.f.)

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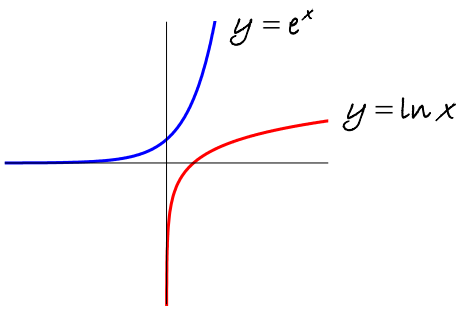
3. (i) $\frac{dy}{dx} = 2e^{2x}$ (ii) $\frac{dy}{dx} = -e^{-x}$ (iii) $\frac{dy}{dx} = -6e^{-3x}$

4.



These curves are reflections of each other in the line $y = x$.

5. (i) $N = 1000e^{0.2t}$

When $t = 2$, $N = 1000e^{0.2\times2} = 1000e^{0.4} = 1491.8$

After 2 hours there are 1492 bacteria.

(ii) When $t = 0$, $N = 1000$.

When number has doubled, $1000e^{0.2t} = 2000$

$e^{0.2t} = 2$

$0.2t = \ln 2$

$t = 5\ln 2 = 3.47$ (3 s.f.)

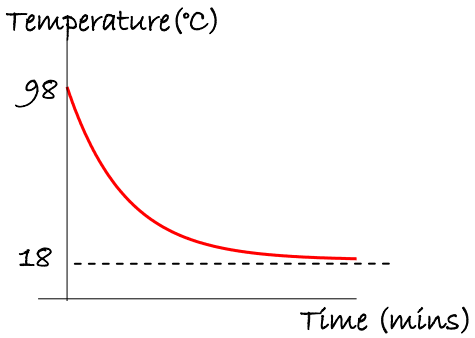
The number of bacteria has doubled after 3.47 hours.

6. (i) $T = 18 + 80e^{-0.5t}$

When $t = 0$, $T = 18 + 80 = 98$

The initial temperature of the liquid is 98°C.

(ii)



(iii) When $t = 10$, $T = 18 + 80e^{-0.5\times10} = 18 + 80e^{-5} = 18.5$

After 10 minutes, the temperature is 18.5°C (3 s.f.)

(iv) When $T = 25$, $18 + 80e^{-0.5t} = 25$

$$80e^{-0.5t} = 7$$

$$e^{-0.5t} = \frac{7}{80}$$

$$-0.5t = \ln\left(\frac{7}{80}\right)$$

$$t = -2\ln\left(\frac{7}{80}\right) = 4.87$$

The temperature is 25°C after 4.87 minutes.

(v) The temperature of the room is 18°C
(the temperature of the liquid approaches this temperature as t becomes large).