Section 1: Exponential functions and logarithms

Solutions to Exercise level 2

1. (i) $\log 12 = \log(2^2 \times 3) = \log 2^2 + \log 3 = 2\log 2 + \log 3$

(ii) $\log\left(\frac{16}{27}\right) = \log\left(\frac{2^4}{3^3}\right) = \log 2^4 - \log 3^3 = 4\log 2 - 3\log 3$

(iii) $\log\sqrt{54} = \log\left(2 \times 3^3\right)^{\frac{1}{2}} = \log 2^{\frac{1}{2}} + \log 3^{\frac{3}{2}} = \frac{1}{2}\log 2 + \frac{3}{2}\log 3$

(iv) $\log\frac{\sqrt{3}}{16} = \log\frac{3^{\frac{1}{2}}}{2^4} = \log 3^{\frac{1}{2}} - \log 2^4 = \frac{1}{2}\log 3 - 4\log 2$

2. (i) $2^x = 18$

$\log 2^x = \log 18$

$x\log 2 = \log 18$

$x = \frac{\log 18}{\log 2} = 4.17$ (3 s.f.)

(ii) $5^x = 100$

$\log 5^x = \log 100$

$x\log 5 = \log 100$

$x = \frac{\log 100}{\log 5} = 2.86$ (3 s.f.)

(iii) $1.5^x = 0.001$

$\log 1.5^x = \log 0.001$

$x\log 1.5 = \log 0.001$

$x = \frac{\log 0.001}{\log 1.5} = -17.0$ (3 s.f.)

(iv) $10^x = 2$

$\log 10^x = \log 2$

$x\log 10 = \log 2$

$x = \frac{\log 2}{\log 10} = 0.301$ (3 s.f.)

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3. (i) $\log_3 x = -\frac{1}{2}$

$\Rightarrow x = 3^{-\frac{1}{2}} = \frac{1}{\sqrt{3}}$

(ii) $2\log x + \log 10 = \log 90$

$\Rightarrow \log x^2 + \log 10 = \log 90$

$\Rightarrow 10x^2 = 90$

$\Rightarrow x^2 = 9$

$\Rightarrow x = 3$

(Note that $x = -3$ does not satisfy the equation, as log (-3) is undefined)

(iii) $\log_{10}\left(\frac{1}{x}\right) = -3$

$\Rightarrow \left(\frac{1}{x}\right) = 10^{-3} = \frac{1}{1000}$

$\Rightarrow x = 1000$

(iv) $\log x + \log y = \frac{1}{2}\log\left(9y^2\right)$

$\Rightarrow \log(xy) = \log(3y)$

$\Rightarrow xy = 3y$

$\Rightarrow x = 3$

4. $P = 500 \times 1.04^t$

(i) (a) After 1 month, $t = \frac{1}{12}$.

$P = 500 \times 1.04^{\frac{1}{12}} = 501.64$

Amount after 1 month = £501.64.

(b) After 6 months, $t = 0.5$

$P = 500 \times 1.04^{0.5} = 509.90$

Amount after 6 months = £509.90.

(c) After 1 year, $t = 1$

$P = 500 \times 1.04^1 = 520$

Amount after 6 months = £520.00.

(d) After 5 years, $t = 5$

$P = 500 \times 1.04^5 = 608.33$

Amount after 6 months = £608.33.

(ii) $500 \times 1.04^t > 800$

$1.04^t > 1.6$

$t \log 1.04 > \log 1.6$

$t > 11.98$

11.98 years = 11 years 11.76 months

It will first exceed £800 after 12 years 0 months.

5. (i) $3^a = 21$

$\Rightarrow a \log 3 = \log 21$

$\Rightarrow a = \dfrac{\log 21}{\log 3} = 2.77$ (3 s.f.)

(ii) $(1.005)^x = 1.1$

$\Rightarrow x \log (1.005) = \log (1.1)$

$\Rightarrow x = \dfrac{\log (1.1)}{\log (1.005)} = 19.1$ (3 s.f.)

(iii) $50^{\frac{1}{a}} = 10$

$\Rightarrow \dfrac{1}{a} \log 50 = \log 10$

$\Rightarrow a = \dfrac{\log 50}{\log 10} \approx 1.699$ (3 d.p.)

(iv) $\left(1 + \dfrac{x}{100}\right)^{12} = 1.25$

$\Rightarrow 12 \log \left(1 + \dfrac{x}{100}\right) = \log (1.25)$

$\Rightarrow \log \left(1 + \dfrac{x}{100}\right) = \dfrac{1}{12} \log (1.25) = 0.0080758\ldots\ldots$

$\Rightarrow 1 + \dfrac{x}{100} = 10^{0.0080758} = 1.018769\ldots\ldots$

$\Rightarrow x = 1.877$ (3 d.p.)

6. The solution in 4 (iv) shows that a monthly interest rate of approximately 1.877% is equivalent to an annual rate of 25% on a debt.