

Section 1: Exponential functions and logarithms

Solutions to Exercise level 1

1. (i) $10^3 = 1000 \Rightarrow \log_{10} 1000 = 3$

(ii) $2^7 = 128 \Rightarrow \log_2 128 = 7$

(iii) $10^{\frac{1}{3}} = \sqrt[3]{10} \Rightarrow \log_{10} \sqrt[3]{10} = \frac{1}{3}$

(iv) $2^{-3} = \frac{1}{8} \Rightarrow \log_2 \frac{1}{8} = -3$

(v) $5^{-\frac{1}{2}} = \frac{1}{\sqrt{5}} \Rightarrow \log_5 \frac{1}{\sqrt{5}} = -\frac{1}{2}$

(vi) $3^{\frac{3}{2}} = \sqrt{27} \Rightarrow \log_3 \sqrt{27} = \frac{3}{2}$

2. (i) $x = \log_2 16 \Rightarrow 2^x = 16 \Rightarrow x = 4$
so $\log_2 16 = 4$

(ii) $x = \log_{10} 1000000 \Rightarrow 10^x = 1000000 \Rightarrow x = 6$
so $\log_{10} 1000000 = 6$

(iii) $x = \log_6 1 \Rightarrow 6^x = 1 \Rightarrow x = 0$
so $\log_6 1 = 0$

(iv) $x = \log_4 \left(\frac{1}{4} \right) \Rightarrow 4^x = \frac{1}{4} \Rightarrow x = -1$
so $\log_4 \left(\frac{1}{4} \right) = -1$

(v) $x = \log_5 \sqrt{5} \Rightarrow 5^x = \sqrt{5} \Rightarrow x = \frac{1}{2}$
so $\log_5 \sqrt{5} = \frac{1}{2}$

(vi) $x = \log_3 \left(\frac{1}{27} \right) \Rightarrow 3^x = \frac{1}{27} \Rightarrow x = -3$
so $\log_3 \left(\frac{1}{27} \right) = -3$

(vii) $x = \log_8 4 \Rightarrow 8^x = 4 \Rightarrow 2^{3x} = 2^2 \Rightarrow x = \frac{2}{3}$
so $\log_8 4 = \frac{2}{3}$

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$$\begin{aligned} \text{(viii)} \quad x &= \log_2 \left(\frac{1}{\sqrt{32}} \right) \Rightarrow 2^x = \frac{1}{\sqrt{32}} = \frac{1}{\sqrt{2^5}} = 2^{-\frac{5}{2}} \Rightarrow x = -\frac{5}{2} \\ \text{so } \log_2 \left(\frac{1}{\sqrt{32}} \right) &= -\frac{5}{2} \end{aligned}$$

3. (i) $\log_2 x = -5 \Rightarrow 2^{-5} = x \Rightarrow x = \frac{1}{32}$

(ii) $\log_3 x = \frac{3}{2} \Rightarrow 3^{\frac{3}{2}} = x \Rightarrow x = \sqrt{27}$

(iii) $\log_x 64 = 2 \Rightarrow x^2 = 64 \Rightarrow x = 8$

(iv) $\log_x \left(\frac{1}{\sqrt{5}} \right) = \frac{1}{2} \Rightarrow x^{\frac{1}{2}} = \frac{1}{\sqrt{5}} \Rightarrow x = \frac{1}{5}$

4. (i) $\log 2 + \log 3 = \log(2 \times 3) = \log 6$

(ii) $\log 10 - \log 2 = \log \frac{10}{2} = \log 5$

(iii) $3 \log 5 = \log 5^3 = \log 125$

(iv) $2 \log 3 - 4 \log 2 = \log 3^2 - \log 2^4 = \log \frac{3^2}{2^4} = \log \frac{9}{16}$

(v) $\frac{1}{2} \log 3 - \frac{1}{4} \log 4 = \log 3^{\frac{1}{2}} - \log 4^{\frac{1}{4}} = \log \frac{\sqrt{3}}{\sqrt{2}} = \log \sqrt{\frac{3}{2}}$

(vi) $2 \log a + 5 \log b - 3 \log c = \log a^2 + \log b^5 - \log c^3 = \log \frac{a^2 b^5}{c^3}$

5. (i) $\begin{aligned} \frac{1}{2} \log 2 - \frac{1}{4} \log 16 &= \log 2^{\frac{1}{2}} - \log 16^{\frac{1}{4}} \\ &= \log \sqrt{2} - \log 2 \\ &= \log \left(\frac{\sqrt{2}}{2} \right) \\ &= \log \left(\frac{1}{\sqrt{2}} \right) \quad (\text{or } -\log \sqrt{2}) \end{aligned}$

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$$\begin{aligned} \text{(ii)} \quad 4 \log_{10} 3 - \log_{10} 9 &= \log_{10} 3^4 - \log_{10} 9 \\ &= \log_{10} \left(\frac{3^4}{9} \right) \\ &= \log_{10} 9 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad 4 \log_3 3 - \log_3 9 &= 4 \log_3 3 - \log_3 3^2 \\ &= 4 \log_3 3 - 2 \log_3 3 \\ &= 4 - 2 = 2 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad 2 \log x + 3 \log y - \log(x^2 y) &= \log x^2 + \log y^3 - \log(x^2 y) \\ &= \log \left(\frac{x^2 y^3}{x^2 y} \right) \\ &= \log y^2 \text{ (or } 2 \log y) \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad a \log b - b \log a + \log a - \log b &= (1 - b) \log a + (a - 1) \log b \\ &= \log a^{1-b} + \log b^{a-1} \\ &= \log(a^{1-b} b^{a-1}) \end{aligned}$$