Section 3: Further integration

Solutions to Exercise level 3

1. (i) $\frac{dy}{dx} = x - \frac{8}{\sqrt{x}}$

$x = 4 \Rightarrow \frac{dy}{dx} = 4 - \frac{8}{2} = 0$, so there is a turning point at $x = 4$.

$\frac{d^2y}{dx^2} = 1 + \frac{4}{x^{\frac{3}{2}}}$, so when $x = 4$, $\frac{d^2y}{dx^2} > 0$

so this is a minimum point.

(ii) $\frac{dy}{dx} = x - \frac{8}{\sqrt{x}}$

$\Rightarrow y = \frac{1}{2}x^2 - 16\sqrt{x} + k$

The curve passes through $(16, 88) \Rightarrow 128 - 64 + k = 88$

$\Rightarrow k = 24$

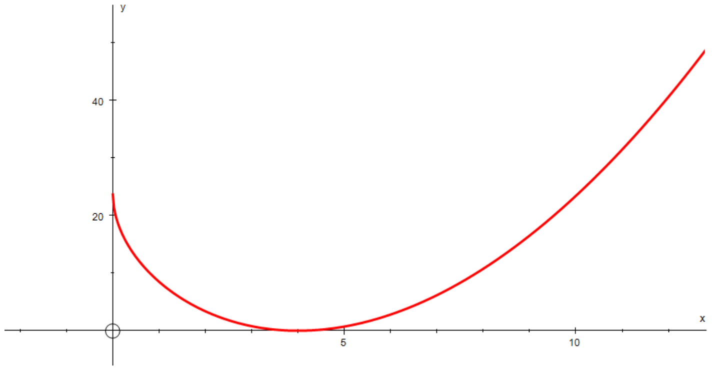
so the equation is $y = \frac{1}{2}x^2 - 16\sqrt{x} + 24$

(iii) The gradient function does not exist for $x \leq 0$.

At the minimum point, $x = 4 \Rightarrow y = 8 - 32 + 24 = 0$, so the minimum point is $(4, 24)$

As x approaches 0, y approaches 24.

So the graph is

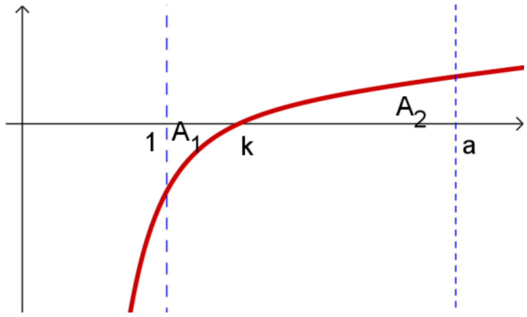


2. $\int_0^1 x^{\frac{1}{n}} = 0.8$

$\left[\frac{x^{\frac{1}{n}+1}}{\frac{1}{n}+1}\right]_0^1 = 0.8$

$$\frac{1}{\frac{1}{n}+1} - 0 = 0.8$$
$$1 = 0.8\left(\tfrac{1}{n}+1\right)$$
$$n = 0.8 + 0.8n$$
$$0.2n = 0.8$$
$$n = 4$$

3. The graph of $y = x - \dfrac{k^3}{x^2}$ crosses the x-axis at $x = k$



The total area is 6.5, and the value of the integral is 1.5,
so $A_1 + A_2 = 6.5$ and $A_2 - A_1 = 1.5$
which gives $A_2 = 4$ and $A_1 = 2.5$

For A_1 (which is below the x-axis),

$$\int_1^k \left(x - \frac{k^3}{x^2}\right) dx = -2.5$$
$$\left[\frac{x^2}{2} + \frac{k^3}{x}\right]_1^k = -2.5$$
$$\tfrac{1}{2}k^2 + k^2 - \tfrac{1}{2} - k^3 = -2.5$$
$$k^3 - \tfrac{3}{2}k^2 - 2 = 0$$
$$2k^3 - 3k^2 - 4 = 0$$

By inspection or by using an equation solver, $k = 2$.

For the whole integral, $\displaystyle\int_1^a \left(x - \frac{8}{x^2}\right) dx = 1.5$

$$\left[\frac{x^2}{2} + \frac{8}{x}\right]_1^a = 1.5$$
$$\tfrac{1}{2}a^2 + \frac{8}{a} - \tfrac{1}{2} - 8 = 1.5$$
$$a^3 - 20a + 16 = 0$$

By inspection or by using an equation solver, $a = 4$.