

Section 3: Further integration

Solutions to Exercise level 2

$$\begin{aligned}
 1. \quad (i) \quad \int (3\sqrt{x} - 2)^2 dx &= \int (3x^{\frac{1}{2}} - 2)^2 dx \\
 &= \int (9x - 12x^{\frac{1}{2}} + 4) dx \\
 &= \frac{9}{2}x^2 - 8x^{\frac{3}{2}} + 4x + c
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad \int \left(\frac{(x-1)^2}{x^5} \right) dx &= \int (x^{-3} - 2x^{-4} + x^{-5}) dx \\
 &= -\frac{1}{2}x^{-2} + \frac{2}{3}x^{-3} - \frac{1}{4}x^{-4} + c
 \end{aligned}$$

$$\begin{aligned}
 (iii) \quad \int \frac{\sqrt{x} + x^{\frac{3}{2}} + 7x^3}{\sqrt{x}} dx &= \int (1 + x + 7x^{\frac{5}{2}}) dx \\
 &= x + \frac{1}{2}x^2 + 2x^{\frac{7}{2}} + c
 \end{aligned}$$

$$\begin{aligned}
 (iv) \quad \int \frac{(x^2-1)(x^2+1)}{x^2} dx &= \int \frac{x^4-1}{x^2} dx \\
 &= \int \left(x^2 - \frac{1}{x^2} \right) dx \\
 &= \frac{1}{3}x^3 + \frac{1}{x} + c
 \end{aligned}$$

$$\begin{aligned}
 2. \quad (i) \quad \int_1^4 \left(\frac{2-x+3x^2}{\sqrt{x}} \right) dx &= \int_1^4 (2x^{-\frac{1}{2}} - x^{\frac{1}{2}} + 3x^{\frac{3}{2}}) dx \\
 &= \left[4x^{\frac{1}{2}} - \frac{2}{3}x^{\frac{3}{2}} + \frac{6}{5}x^{\frac{5}{2}} \right]_1^4 \\
 &= \left(4(4)^{\frac{1}{2}} - \frac{2}{3}(4)^{\frac{3}{2}} + \frac{6}{5}(4)^{\frac{5}{2}} \right) - \left(4(1)^{\frac{1}{2}} - \frac{2}{3}(1)^{\frac{3}{2}} + \frac{6}{5}(1)^{\frac{5}{2}} \right) \\
 &= \left(8 - \frac{16}{3} + \frac{192}{5} \right) - \left(4 - \frac{2}{3} + \frac{6}{5} \right) \\
 &= \frac{616}{15} - \frac{68}{15} \\
 &= \frac{548}{15}
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad \int_1^2 \left(\frac{x^2-1}{x^4} \right) dx &= \int_1^2 ((x^2-1)(x^{-4})) dx \\
 &= \int_1^2 (x^{-2} - x^{-4}) dx \\
 &= \left[-x^{-1} + \frac{1}{3}x^{-3} \right]_1^2
 \end{aligned}$$

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$$\begin{aligned} &= \left(-(2)^{-1} + \frac{1}{3}(2)^{-3} \right) - \left(-(1)^{-1} + \frac{1}{3}(1)^{-3} \right) \\ &= \left(-\frac{1}{2} + \frac{1}{24} \right) - \left(-1 + \frac{1}{3} \right) \\ &= -\frac{11}{24} - \left(-\frac{2}{3} \right) \\ &= \frac{5}{24} \end{aligned}$$

3. $\frac{dy}{dx} = \frac{x-3}{x^3} = x^{-2} - 3x^{-3}$

$$y = \int (x^{-2} - 3x^{-3}) dx$$

$$y = -x^{-1} + \frac{3}{2}x^{-2} + c$$

When $x=1$, $y=1$

$$1 = -(1)^{-1} + \frac{3}{2}(1)^{-2} + c$$

$$1 = -1 + \frac{3}{2} + c$$

$$c = \frac{1}{2}$$

$$y = -\frac{1}{x} + \frac{3}{2x^2} + \frac{1}{2}$$

4. $y = x - \sqrt{x}$

The graph meets the x-axis when $y=0$.

$$y = x(1 - x^{-\frac{1}{2}})$$

$$1 - x^{-\frac{1}{2}} = 0 \quad \text{or} \quad x = 0$$

$$1 = x^{-\frac{1}{2}}$$

$$x = 1$$

$$\begin{aligned} \int_0^1 (x - x^{\frac{1}{2}}) dx &= \left[\frac{1}{2}x^2 - \frac{2}{3}x^{\frac{3}{2}} \right]_0^1 \\ &= \left(\frac{1}{2}(1)^2 - \frac{2}{3}(1)^{\frac{3}{2}} \right) - \left(\frac{1}{2}(0)^2 - \frac{2}{3}(0)^{\frac{3}{2}} \right) \\ &= \left(\frac{1}{2} - \frac{2}{3} \right) - 0 \\ &= -\frac{1}{6} \end{aligned}$$

Area cannot be negative so area = $\frac{1}{6}$