

## Section 3: Further integration

## Solutions to Exercise level 1

$$\begin{aligned}
 1. \quad (i) \quad \int \frac{1}{x^2} dx &= \int x^{-2} dx \\
 &= -x^{-1} + c \\
 &= -\frac{1}{x} + c
 \end{aligned}$$

$$(ii) \quad \int x^{\frac{4}{5}} dx = \frac{5}{9} x^{\frac{9}{5}} + c$$

$$\begin{aligned}
 (iii) \quad \int \sqrt[3]{x} dx &= \int x^{\frac{1}{3}} dx \\
 &= \frac{3}{4} x^{\frac{4}{3}} + c
 \end{aligned}$$

$$(iv) \quad \int (2x^{\frac{3}{7}} - 3x^{\frac{2}{5}}) dx = \frac{14}{9} x^{\frac{10}{7}} - \frac{9}{5} x^{\frac{5}{5}} + c$$

$$(v) \quad \int (3x^{-3} - 4x^{-4}) dx = -\frac{3}{2} x^{-2} + \frac{4}{3} x^{-3} + c$$

$$\begin{aligned}
 (vi) \quad \int \left( \frac{1}{x^2} - \frac{2}{x^3} \right) dx &= -\frac{1}{x} + \frac{1}{x^2} + c \\
 &= \frac{1-x}{x^2} + c
 \end{aligned}$$

$$\begin{aligned}
 2. \quad (i) \quad \int_1^3 \frac{1}{x^3} dx &= \int_1^3 x^{-3} dx \\
 &= \left[ -\frac{1}{2} x^{-2} \right]_1^3 \\
 &= -\frac{1}{2} (3)^{-2} - \left( -\frac{1}{2} (1)^{-2} \right) \\
 &= -\frac{1}{18} + \frac{1}{2} \\
 &= \frac{4}{9}
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad \int_1^9 \frac{1}{\sqrt{x}} dx &= \int_1^9 x^{-\frac{1}{2}} dx \\
 &= \left[ 2x^{\frac{1}{2}} \right]_1^9 \\
 &= (2(9)^{\frac{1}{2}}) - (2(1)^{\frac{1}{2}}) \\
 &= 6 - 2 \\
 &= 4
 \end{aligned}$$

## Edexcel AS Maths Integration 3 Exercise solutions

$$\begin{aligned} \text{(iii)} \int_1^4 (\sqrt{x} - 1) dx &= \left[ \frac{2}{3} x^{\frac{3}{2}} - x \right]_1^4 \\ &= \left( \frac{16}{3} - 4 \right) - \left( \frac{2}{3} - 1 \right) \\ &= \frac{5}{3} \end{aligned}$$

$$\begin{aligned} \text{(iv)} \int_1^3 \frac{1}{x^2} - \frac{1}{x^3} dx &= \left[ -\frac{1}{x} + \frac{1}{2x^2} \right]_1^3 \\ &= \left( -\frac{1}{3} + \frac{1}{18} \right) - \left( -1 + \frac{1}{2} \right) \\ &= \frac{2}{9} \end{aligned}$$

$$3. \frac{dy}{dx} = 2\sqrt{x} - 3x = 2x^{\frac{1}{2}} - 3x$$

$$y = \int (2x^{\frac{1}{2}} - 3x) dx$$

$$y = \frac{4}{3} x^{\frac{3}{2}} - \frac{3}{2} x^2 + c$$

$$\text{When } x=1, y=-1$$

$$-1 = \frac{4}{3}(1)^{\frac{3}{2}} - \frac{3}{2}(1)^2 + c$$

$$-1 = \frac{4}{3} - \frac{3}{2} + c$$

$$-1 = -\frac{1}{6} + c$$

$$c = -\frac{5}{6}$$

$$y = \frac{4}{3} x^{\frac{3}{2}} - \frac{3}{2} x^2 - \frac{5}{6}$$

$$\begin{aligned} 4. \int_1^4 \left( \frac{1}{x^2} + x \right) dx &= \int_1^4 (x^{-2} + x) dx \\ &= \left[ -x^{-1} + \frac{1}{2} x^2 \right]_1^4 \\ &= \left( -(4)^{-1} + \frac{1}{2}(4)^2 \right) - \left( -(1)^{-1} + \frac{1}{2}(1)^2 \right) \\ &= \left( -\frac{1}{4} + 8 \right) - \left( -1 + \frac{1}{2} \right) \\ &= \frac{31}{4} + \frac{1}{2} \\ &= \frac{33}{4} \end{aligned}$$