Section 2: Finding the area under a curve

Solutions to Exercise level 1

1. (i) $\int 4x^3 dx = x^4 + c$

(ii) $\int (x^3 - 3x^2) dx = \frac{1}{4}x^4 - x^3 + c$

(iii) $\int (10x^4 + 3x^2 + 4) dx = 2x^5 + x^3 + 4x + c$

(iv) $\int (3x-1)^2 dx = \int \left(9x^2 - 6x + 1\right) dx$
$= 3x^3 - 3x^2 + x + c$

(v) $\int x(3x-4)\, dx = \int \left(3x^2 - 4x\right) dx$
$= x^3 - 2x^2 + c$

2. (i) $\int_{-1}^{1} (4x+5) dx = \left[2x^2 + 5x\right]_{-1}^{1}$
$= 2 + 5 - (2 - 5)$
$= 7 - (-3)$
$= 10$

(ii) $\int_{-1}^{0} (6x^2 - 2x) dx = \left[2x^3 - x^2\right]_{-1}^{0}$
$= 0 - (-2 - 1)$
$= 3$

(iii) $\int_{2}^{4} (x^2 - x + 3) dx = \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 + 3x\right]_{2}^{4}$
$= \left(\frac{64}{3} - 8 + 12\right) - \left(\frac{8}{3} - 2 + 6\right)$
$= \frac{64}{3} + 4 - \frac{8}{3} - 4$
$= \frac{56}{3}$

(iv) $\int_{-1}^{2} (2 + x - x^2) dx = \left[2x + \frac{1}{2}x^2 - \frac{1}{3}x^3\right]_{-1}^{2}$
$= \left(4 + 2 - \frac{8}{3}\right) - \left(-2 + \frac{1}{2} + \frac{1}{3}\right)$
$= 6 - \frac{8}{3} + 2 - \frac{1}{2} - \frac{1}{3}$
$= 8 - 3 - \frac{1}{2}$
$= \frac{9}{2}$

(v) $\int_{-1}^{2}(x^3 - x + 4)dx = \left[\frac{1}{4}x^4 - \frac{1}{2}x^2 + 4x\right]_{-1}^{2}$

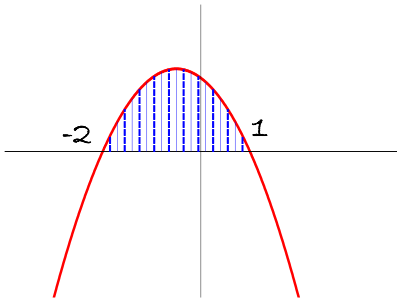
$= (4 - 2 + 8) - (\frac{1}{4} - \frac{1}{2} - 4)$

$= 10 + \frac{1}{4} + 4$

$= 14.25$

3. (i) $y = (1 - x)(x + 2)$

The graph cuts the x-axis at $x = 1$ and $x = -2$.
The coefficient of x^2 is negative, so the graph is "upside down".



$\text{Area} = \int_{-2}^{1}(1 - x)(x + 2)dx$

$= \int_{-2}^{1}(2 - x - x^2)dx$

$= \left[2x - \frac{1}{2}x^2 - \frac{1}{3}x^3\right]_{-2}^{1}$

$= (2 - \frac{1}{2} - \frac{1}{3}) - (-4 - 2 + \frac{8}{3})$

$= 2 - \frac{1}{2} - \frac{1}{3} + 6 - \frac{8}{3}$

$= \frac{9}{2}$ square units

(ii) $y = 3x^2 - x^3 = x^2(3 - x)$

The graph cuts the x-axis at (3, 0) and touches the x-axis at the origin.

$\text{Area} = \int_{0}^{3}(3x^2 - x^3)dx$

$= \left[x^3 - \frac{1}{4}x^4\right]_{0}^{3}$

$= 27 - \frac{81}{4} - 0$

$= 6.75$ square units