

Section 1: Introduction to integration

Solutions to Exercise level 3 (Extension)

1. (i)  $\frac{dy}{dx} = (x-2)(x+5) = x^2 + 3x - 10$

$$\Rightarrow y = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 10x + k$$

The curve passes through (0, 3), so  $k = 3$

so the equation of the graph is  $y = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 10x + 3$

(ii) As before,  $y = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 10x + k$

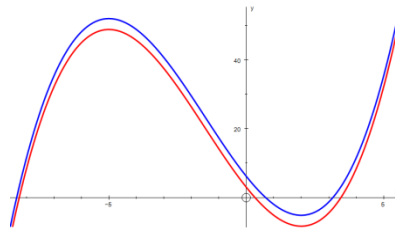
The curve passes through (1, -2), so  $\frac{1}{3} + \frac{3}{2} - 10 + k = -2$

$$\Rightarrow k = \frac{37}{6}$$

so the equation of the graph is  $y = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 10x + \frac{37}{6}$

The two curves in (i) and (ii) have the same shape, and the curve in (ii) is obtained from that in (i) by a translation parallel to the y-axis through

$$\begin{pmatrix} 0 \\ \frac{19}{6} \end{pmatrix}.$$



2.  $\frac{dy}{dx} = 3x^2 + 3x + a \Rightarrow y = x^3 + \frac{3}{2}x^2 + ax + c_1$

Passes through (1, a) so  $a = 1 + \frac{3}{2} + a + c_1$

$$\Rightarrow c_1 = -\frac{5}{2}$$

so graph is  $y = x^3 + \frac{3}{2}x^2 + ax - \frac{5}{2}$

$$\frac{dy}{dx} = 3x^2 - 2x + 1 \Rightarrow y = x^3 - x^2 + x + c_2$$

Passes through (1, a) so  $a = 1 - 1 + 1 + c_2$

$$\Rightarrow c_2 = a - 1$$

so graph is  $y = x^3 - x^2 + x + a - 1$

The graphs also meet at  $x = -2$ ,

## Edexcel AS Maths Integration 1 Exercise solutions

$$\text{so } -8 + \frac{3}{2} \times 4 - 2a - \frac{5}{2} = -8 - 4 - 2 + a - 1$$

$$10.5 = 3a$$

$$a = 3.5$$

$$\text{Equations of graphs are } y = x^3 + \frac{3}{2}x^2 + \frac{7}{2}x - \frac{5}{2}$$

$$\text{and } y = x^3 - x^2 + x + \frac{5}{2}$$

3. The turning points are the roots of the gradient function, which is quadratic.

$$\frac{dy}{dx} = k(x+1)(x-2) = k(x^2 - x - 2)$$

$$y = k\left(\frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x\right) + c$$

$$\text{Passes through } (2, 1) \text{ so } 1 = k\left(\frac{8}{3} - 2 - 4\right) + c \Rightarrow 1 = -\frac{10}{3}k + c$$

$$\text{Passes through } (-1, -2) \text{ so } -2 = k\left(-\frac{1}{3} - \frac{1}{2} + 2\right) + c \Rightarrow -2 = \frac{7}{6}k + c$$

$$\text{Subtracting gives } 3 = -\frac{9}{2}k \Rightarrow k = -\frac{2}{3}$$

$$c = 1 - \frac{20}{9} = -\frac{11}{9}$$

$$\text{The equation of the graph is } y = -\frac{2}{3}\left(\frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x\right) - \frac{11}{9}$$

$$y = -\frac{2}{9}x^3 + \frac{1}{3}x^2 + \frac{4}{3}x - \frac{11}{9}$$