Section 1: Introduction to integration

Solutions to Exercise level 2

1. (i) $\frac{dy}{dx} = 4x^2 + x$

$y = \frac{4}{3}x^3 + \frac{1}{2}x^2 + c$

When $x = 1$, $y = 2$

$2 = \frac{4}{3} \times 1^3 + \frac{1}{2} \times 1^2 + c$

$c = 2 - \frac{4}{3} - \frac{1}{2} = \frac{1}{6}$

$y = \frac{4}{3}x^3 + \frac{1}{2}x^2 + \frac{1}{6}$

(ii) When $x = 3$, $y = \frac{4}{3} \times 3^3 + \frac{1}{2} \times 3^2 + \frac{1}{6}$

$= 36 + \frac{9}{2} + \frac{1}{6}$

$= 40\frac{2}{3}$

2. $\frac{dy}{dx} = 4(1 - x)$

At maximum point, $\frac{dy}{dx} = 0 \quad \Rightarrow x = 1$

So the curve passes through the point (1, 8).

$\frac{dy}{dx} = 4(1 - x) = 4 - 4x$

$y = 4x - 2x^2 + c$

When $x = 1$, $y = 8$

$8 = 4 - 2 + c \Rightarrow c = 6$

The equation of the curve is $y = 4x - 2x^2 + 6$.

3. $\frac{dy}{dx} = (x - 1)(3x - 5) = 3x^2 - 8x + 5$

$y = 3 \times \frac{1}{3}x^3 - 8 \times \frac{1}{2}x^2 + 5x + c$

$= x^3 - 4x^2 + 5x + c$

When $x = 1$, $y = 2$

$2 = 1^3 - 4 \times 1^2 + 5 \times 1 + c$

$c = 2 - 1 + 4 - 5 = 0$

$y = x^3 - 4x^2 + 5x$

4. At turning points, $4x^2 - 1 = 0$

$$(2x-1)(2x+1) = 0$$

$$x = \tfrac{1}{2} \text{ or } -\tfrac{1}{2}$$

For $x < -\tfrac{1}{2}$, $\dfrac{dy}{dx} > 0$

For $-\tfrac{1}{2} < x < \tfrac{1}{2}$, $\dfrac{dy}{dx} < 0$

For $x > \tfrac{1}{2}$, $\dfrac{dy}{dx} > 0$

Therefore there is a maximum point where $x = -\tfrac{1}{2}$ and a minimum point where $x = \tfrac{1}{2}$.

So the graph passes through the point $\left(\tfrac{1}{2}, 1\right)$.

$$\frac{dy}{dx} = 4x^2 - 1$$

$$y = \tfrac{4}{3}x^3 - x + c$$

When $x = \tfrac{1}{2}$, $y = 1$ $\Rightarrow 1 = \tfrac{4}{3}\left(\tfrac{1}{2}\right)^3 - \tfrac{1}{2} + c$

$$\Rightarrow 1 = \tfrac{1}{6} - \tfrac{1}{2} + c$$

$$\Rightarrow c = \tfrac{4}{3}$$

The equation of the curve is $y = \tfrac{4}{3}x^3 - x + \tfrac{4}{3}$.

The maximum point is when $x = -\tfrac{1}{2}$.

$$y = \tfrac{4}{3}\left(-\tfrac{1}{2}\right)^3 - \left(-\tfrac{1}{2}\right) + \tfrac{4}{3}$$

$$= -\tfrac{1}{6} + \tfrac{1}{2} + \tfrac{4}{3}$$

$$= \tfrac{5}{3}$$

5. (i) For $x = -2$, $\dfrac{dy}{dx} = 0$ $\Rightarrow 12 + 4 + k = 0$

$$\Rightarrow k = -16$$

and so $\dfrac{dy}{dx} = 3x^2 - 2x - 16$

(ii) $\dfrac{dy}{dx} = 3x^2 - 2x + 8 \Rightarrow y = x^3 - x^2 - 16x + c$

and since the curve passes through (1, 3)

$3 = 1 - 1 - 16 + c \Rightarrow c = 19$

so the equation of the curve is $y = x^3 - x^2 - 16x + 19$