Section 4: More about differentiation

Solutions to Exercise level 1

1. (i) $y = x^3 - 3x^2 + 4x - 1$

$$\frac{dy}{dx} = 3x^2 - 6x + 4$$

$$\frac{d^2y}{dx^2} = 6x - 6$$

(ii) $y = \frac{1}{x} - \frac{2}{x^2} = x^{-1} - 2x^{-2}$

$$\frac{dy}{dx} = -x^{-2} + 4x^{-3}$$

$$\frac{d^2y}{dx^2} = -2x^{-3} - 12x^{-4}$$

(iii) $y = 2\sqrt{x} = 2x^{\frac{1}{2}}$

$$\frac{dy}{dx} = x^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = -\tfrac{1}{2}x^{-\frac{3}{2}} = -\frac{1}{2x\sqrt{x}}$$

2. (i) $y = x^3 - 3x^2 + 6$

$$\frac{dy}{dx} = 3x^2 - 6x$$

$$\frac{d^2y}{dx^2} = 6x - 6$$

(ii) At turning points, $\frac{dy}{dx} = 0$

$$3x^2 - 6x = 0$$

$$3x(x-2) = 0$$

$$x = 0 \text{ or } x = 2$$

When $x = 0$, $y = 6$

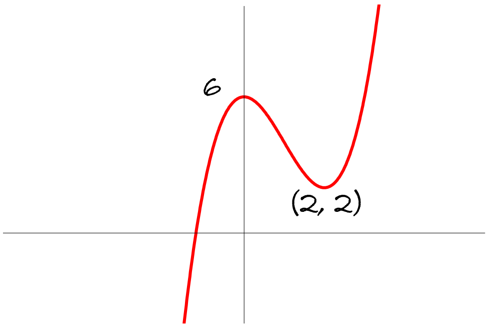
When $x = 2$, $y = 2^3 - 3\times 2^2 + 6 = 8 - 12 + 6 = 2$

The turning points are $(0, 6)$ and $(2, 2)$.

When $x = 0$, $\frac{d^2y}{dx^2} = 0 - 6 < 0$, so $(0, 6)$ is a maximum point.

When $x = 2$, $\frac{d^2y}{dx^2} = 12 - 6 > 0$, so (2, 2) is a minimum point.

(iii)



3. (i) $x + 2y = 100 \Rightarrow y = \frac{1}{2}(100 - x)$

$$A = xy$$
$$= \frac{1}{2}x(100 - x)$$

(ii) $A = 50x - \frac{1}{2}x^2$

$$\Rightarrow \frac{dA}{dx} = 50 - x$$
$$\Rightarrow \frac{d^2A}{dx^2} = -1$$

(iii) At maximum, $\frac{dA}{dx} = 0 \Rightarrow x = 50$

and $\frac{d^2A}{dx^2} < 0$, so $x = 50$ gives a maximum

and the area $= 50 \times 25 = 1250$

4. $y = x^2 - 3x + 1$

When $x = 1$, $y = -1$

When $x = 1 + h$, $y = (1+h)^2 - 3(1+h) + 1$

$$= 1 + 2h + h^2 - 3 - 3h + 1$$
$$= h^2 - h - 1$$

Gradient of chord $= \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{h^2 - h - 1 - (-1)}{1 + h - 1}$$
$$= \frac{h^2 - h}{h}$$
$$= h - 1$$

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5. (i) $y = 2x^2 - x - 1$

When $x = 1$, $y = 0$

When $x = 1 + h$, $y = 2(1+h)^2 - (1+h) - 1$

$$= 2 + 4h + 2h^2 - 1 - h - 1$$

$$= 2h^2 + 3h$$

Gradient of chord $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{2h^2 + 3h - 0}{1 + h - 1}$$

$$= \frac{2h^2 + 3h}{h}$$

$$= 2h + 3$$

(ii) As $h \to 0$, gradient of chord $\to 3$.

So the gradient of the tangent at P is 3.