Section 2: Maximum and minimum points

Solutions to Exercise level 2

1. $f(x) = x^3 + x^2 - x + 3$

   $f'(x) = 3x^2 + 2x - 1$

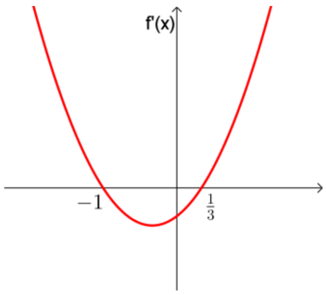
   When f(x) is an increasing function, $f'(x) > 0$

   $\Rightarrow 3x^2 + 2x - 1 > 0$

   $\Rightarrow (3x-1)(x+1) > 0$

   $\Rightarrow x > \frac{1}{3}$ or $x < -1$

   So f(x) is increasing for $x < -1$ and $x > \frac{1}{3}$.



2. $f(x) = x^3 - 6x^2 + 9x + 5$

   $f'(x) = 3x^2 - 12x + 9$

   When f(x) is a decreasing function, $f'(x) < 0$

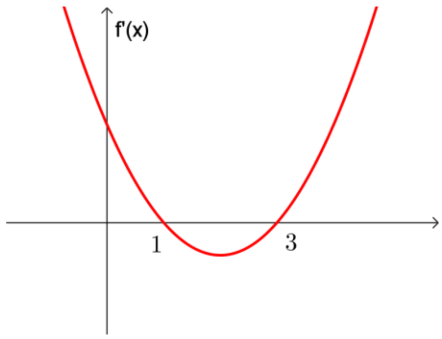
   $\Rightarrow 3x^2 - 12x + 9 < 0$

   $\Rightarrow x^2 - 4x + 3 < 0$

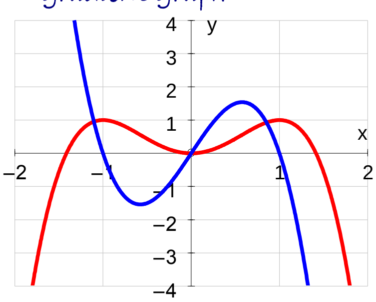
   $\Rightarrow (x-1)(x-3) > 0$

   $\Rightarrow 1 < x < 3$

   So f(x) is decreasing for $1 < x < 3$.



3. gradient graph



4. (i) $y = 2x + x^2 - 4x^3$

$\frac{dy}{dx} = 2 + 2x - 12x^2$

At turning points, $\frac{dy}{dx} = 0$

$2 + 2x - 12x^2 = 0$

$1 + x - 6x^2 = 0$

$6x^2 - x - 1 = 0$

$(3x+1)(2x-1) = 0$

$x = -\frac{1}{3}$ or $x = \frac{1}{2}$

When $x = -\frac{1}{3}$, $y = 2\left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right)^2 - 4\left(-\frac{1}{3}\right)^3 = -\frac{2}{3} + \frac{1}{9} + \frac{4}{27} = \frac{-18+3+4}{27} = -\frac{11}{27}$

When $x = \frac{1}{2}$, $y = 2\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right)^3 = 1 + \frac{1}{4} - \frac{1}{2} = \frac{3}{4}$

The turning points are $\left(-\frac{1}{3}, -\frac{11}{27}\right)$ and $\left(\frac{1}{2}, \frac{3}{4}\right)$.

x	$x < -\frac{1}{3}$	$x = -\frac{1}{3}$	$-\frac{1}{3} < x < \frac{1}{2}$	$x = \frac{1}{2}$	$x > \frac{1}{2}$
$\frac{dy}{dx}$	-ve ╲	0 —	+ve ╱	0 —	-ve ╲

$\left(-\frac{1}{3}, -\frac{11}{27}\right)$ is a minimum point.

$\left(\frac{1}{2}, \frac{3}{4}\right)$ is a maximum point.

(ii) $y = 2x + x^2 - 4x^3$

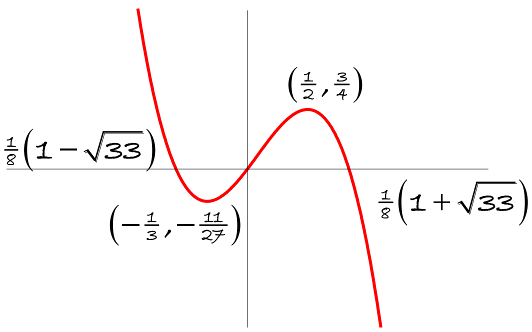
$= x(2 + x - 4x^2)$

$= -x(4x^2 - x - 2)$

The curve cuts the x-axis at $x = 0$ and at the points satisfying $4x^2 - x - 2 = 0$.

For this quadratic equation, $a = 4, b = -1, c = -2$

Using the quadratic formula, $x = \frac{1 \pm \sqrt{1 - 4 \times 4 \times -2}}{8} = \frac{1 \pm \sqrt{33}}{8}$



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5. $y = x^3 + px^2 + q$

$\frac{dy}{dx} = 3x^2 + 2px$

At turning points, $\frac{dy}{dx} = 0$

$3x^2 + 2px = 0$

$x(3x + 2p) = 0$

$x = 0$ or $x = -\frac{2p}{3}$

Since there is a minimum point at $x = 4$, $-\frac{2p}{3} = 4 \quad \Rightarrow p = -6$

The curve is therefore $y = x^3 - 6x^2 + q$.

The point (4, -11) lies on the curve, so $-11 = 4^3 - 6 \times 4^2 + q$

$-11 = 64 - 96 + q$

$q = 21$

The equation of the curve is $y = x^3 - 6x^2 + 21$.

The other turning point is at $x = 0$, so the maximum point is (0, 21).

6. (i) $y = x^3 + ax^2 + bx + c$

The graph passes through the point (1, 1)

so $1 = 1 + a + b + c$

$a + b + c = 0$

(ii) $\frac{dy}{dx} = 3x^2 + 2ax + b$

Turning points are when $3x^2 + 2ax + b = 0$

There is a turning point when $x = -1$, so $3(-1)^2 + 2a \times -1 + b = 0$

$3 - 2a + b = 0$

$2a - b = 3$

There is a turning point when $x = 3$, so $3 \times 3^2 + 2a \times 3 + b = 0$

$27 + 6a + b = 0$

$6a + b = -27$

(iii) $a + b + c = 0$ (1)

$2a - b = 3$ (2)

$6a + b = -27$ (3)

Adding (2) and (3): $8a = -24 \quad \Rightarrow a = -3$

Substituting into (2) gives: $b = 2a - 3 = -6 - 3 = -9$

Substituting into (1) gives: $c = -a - b = 9 + 3 = 12$

$a = -3, b = -9, c = 12$