Section 2: Maximum and minimum points

Solutions to Exercise level 1

1. $f(x) = 2x^2 - 3x + 1$

   $f'(x) = 4x - 3$

   When $f(x)$ is increasing, $f'(x) > 0$

   $\Rightarrow 4x - 3 > 0$

   $\Rightarrow 4x > 3$

   $\Rightarrow x > \frac{3}{4}$

2. $f(x) = 4 + 7x - 3x^2$

   $f'(x) = 7 - 6x$

   When $f(x)$ is decreasing, $f'(x) < 0$

   $\Rightarrow 7 - 6x < 0$

   $\Rightarrow 7 < 6x$

   $\Rightarrow 6x > 7$

   $\Rightarrow x > \frac{7}{6}$

3. The gradient of f(x) starts as negative, becomes zero and then becomes positive. This could be either C or D, but in C the gradient is zero when $x = 0$, so it must be D.

   The gradient of g(x) starts as positive, is zero when $x = 0$ and then becomes positive. This is graph B.

   The gradient of p(x) is a constant positive value. This is graph A.

   The gradient of q(x) starts as negative, becomes zero when $x = 0$, and then becomes positive. This is graph C.

4. (i) $y = x^3 + 6x^2 + 9x$

   $\frac{dy}{dx} = 3x^2 + 12x + 9$

(ii) $\frac{dy}{dx} = 0$

$3x^2 + 12x + 9 = 0$

$x^2 + 4x + 3 = 0$

$(x+1)(x+3) = 0$

$x = -1$ or $x = -3$

When $x = -1$, $y = (-1)^3 + 6(-1)^2 + 9 \times -1 = -1 + 6 - 9 = -4$

When $x = -3$, $y = (-3)^3 + 6(-3)^2 + 9 \times -3 = -27 + 54 - 27 = 0$

The turning points are (-1, -4) and (-3, 0)

(iii)

x	$x < -3$	$x = -3$	$-3 < x < -1$	$x = -1$	$x > -1$
$\frac{dy}{dx}$	+ve /	0 —	-ve \	0 —	+ve /

The point (-3, 0) is a maximum point.
The point (-1, -4) is a minimum point.

(iv) $y = x^3 + 6x^2 + 9x$

$= x(x^2 + 6x + 9)$

$= x(x+3)^2$

The graph cuts the x-axis at $x = 0$ and $x = -3$ (repeated).
The graph cuts the y-axis at $y = 0$.