Section 3: Extending the rule

Solutions to Exercise level 2

1. (i) $\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{3}{2}}$

$= -\frac{1}{2x^{\frac{3}{2}}}$

(ii) $\frac{dy}{dx} = -(-4)x^{-5}$

$= \frac{4}{x^5}$

(iii) $y = (x^2 - 4x + 4)\sqrt{x}$

$= x^{\frac{5}{2}} - 4x^{\frac{3}{2}} + 4x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}$

$= \frac{1}{2}x^{\frac{1}{2}}\left(5x - 12 + \frac{2}{x}\right)$

(iv) $y = \frac{x^2 + 2x + 1}{\sqrt{x}}$

$= x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + x^{-\frac{1}{2}}$

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} + x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$

$= \frac{1}{2}x^{\frac{1}{2}}\left(3 + \frac{2}{x} - \frac{1}{x^2}\right)$

2. When $y = 0$, $x^2 + \frac{1}{x} = 0 \Rightarrow x^3 = -1 \Rightarrow x = -1$

so P = (-1, 0).

$\frac{dy}{dx} = 2x - \frac{1}{x^2}$

so when $x = -1$, $\frac{dy}{dx} = -3$

Equation of tangent through (-1, 0) is $y - 0 = -3(x - (-1))$

$\Rightarrow y = -3x - 3$

Gradient of normal at (-1, 0) is $\frac{1}{3}$

Equation of normal through (-1, 0) is $y-0=\frac{1}{3}(x-(-1))$

$\Rightarrow 3y = x+1$

3. (i) $y = x - \frac{4}{x^2} = x - 4x^{-2}$

$\frac{dy}{dx} = 1 + 8x^{-3}$

Stationary points occur when $\frac{dy}{dx} = 0$.

$0 = 1 + 8x^{-3}$

$-1 = 8x^{-3}$

$x^3 = -8$

$x = -2$

When $x = -2$, $y = -3$

$\frac{d^2y}{dx^2} = -24x^{-4}$

At $x = -2$, $\frac{d^2y}{dx^2} = -24(-2)^{-4} = -1.5$

As this is negative, $(-2, -3)$ is a local maximum.

(ii) $y = \sqrt{x} + \frac{1}{\sqrt{x}} = x^{\frac{1}{2}} + x^{-\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$

Stationary points occur when $\frac{dy}{dx} = 0$.

$0 = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$

$\frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2}x^{-\frac{3}{2}}$

$x^{-\frac{1}{2}} = x^{-\frac{3}{2}}$

$1 = x^{-1}$

$x = 1$

When $x = 1$, $y = 2$

$\frac{d^2y}{dx^2} = -\frac{1}{4}x^{-\frac{3}{2}} + \frac{3}{4}x^{-\frac{5}{2}}$

At $x = 1$, $\frac{d^2y}{dx^2} = -\frac{1}{4} + \frac{3}{4} = \frac{1}{2}$

As this is positive, $(1, 2)$ is a local minimum.

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4. The coordinates of P are (1, 2).

$$y = \frac{2}{x} \quad \Rightarrow \frac{dy}{dx} = -\frac{2}{x^2}$$

At P, gradient of tangent $= -\frac{2}{1^2} = -2$ and so gradient of normal $= -\frac{1}{2}$.

Equation of normal at P is $y - 2 = \frac{1}{2}(x-1)$

$$y = \frac{1}{2}x + \frac{3}{2}$$

At Q, this normal meets the curve again, so $\frac{1}{2}x + \frac{3}{2} = \frac{2}{x}$

$$x^2 + 3x = 4$$

$$x^2 + 3x - 4 = 0$$

$$(x-1)(x+4) = 0$$

so $x = 1$ (which is P) or $x = -4$ (which is Q).

Coordinates of Q are $(-4, -\frac{1}{2})$

Gradient of tangent at P is -2 so equation of tangent is $y - 2 = -2(x-1)$

$$y = -2x + 4$$

Gradient of tangent at Q is $-\frac{1}{8}$ so equation of tangent is $y + \frac{1}{2} = -\frac{1}{8}(x+4)$

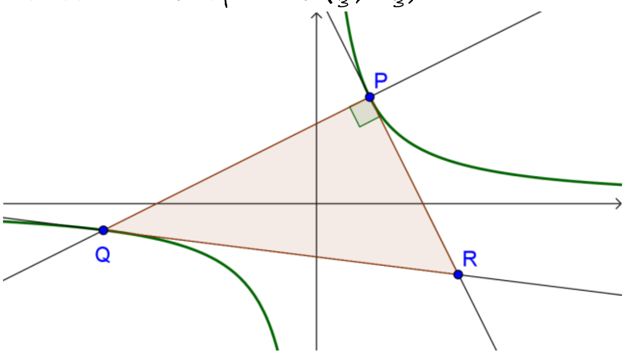
$$y = -\frac{1}{8}x - 1$$

R is where P and Q intersect: $-2x + 4 = -\frac{1}{8}x - 1$

$$5 = \frac{15}{8}x$$

$$x = \frac{8}{3}, y = -\frac{4}{3}$$

So coordinates of R are $(\frac{8}{3}, -\frac{4}{3})$



Since PQ is the normal at P and PR is the tangent at P, the triangle is right-angled .

$$PQ = \sqrt{(1+4)^2 + (2+\frac{1}{2})^2} = \sqrt{25 + \frac{25}{4}} = \sqrt{\frac{125}{4}} = \frac{5}{2}\sqrt{5}$$

$$PR = \sqrt{(1-\frac{8}{3})^2 + (2+\frac{4}{3})^2} = \sqrt{\frac{25}{9} + \frac{100}{9}} = \sqrt{\frac{125}{9}} = \frac{5}{3}\sqrt{5}$$

Area of triangle $= \frac{1}{2} \times \frac{5}{2}\sqrt{5} \times \frac{5}{3}\sqrt{5}$

$$= \frac{125}{12}$$