

Section 3: Extending the rule

Solutions to Exercise level 1

$$1. \text{ (i) } y = \frac{1}{x^3} = x^{-3}$$

$$\frac{dy}{dx} = -3x^{-4} = -\frac{3}{x^4}$$

$$\text{(ii) } y = \sqrt[3]{x} = x^{\frac{1}{3}}$$

$$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$$

$$\text{(iii) } y = \frac{2}{x} - \frac{3}{x^2} = 2x^{-1} - 3x^{-2}$$

$$\frac{dy}{dx} = -2x^{-2} + 6x^{-3} = -\frac{2}{x^2} + \frac{6}{x^3}$$

$$\text{(iv) } y = 4\sqrt{x} - \frac{3}{\sqrt{x}} = 4x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = 2x^{-\frac{1}{2}} + \frac{3}{2}x^{-\frac{3}{2}}$$

$$\text{(v) } y = 3x^{-5} - 2x^{-7}$$

$$\frac{dy}{dx} = -15x^{-6} + 14x^{-8} = -\frac{15}{x^6} + \frac{14}{x^8}$$

$$\text{(vi) } y = 2x^{\frac{2}{3}} - 5x^{-\frac{2}{3}}$$

$$\frac{dy}{dx} = \frac{4}{3}x^{-\frac{1}{3}} + \frac{10}{3}x^{-\frac{5}{3}}$$

$$\text{(vii) } y = \frac{x^2 - 2x + 3}{2x^2} = \frac{1}{2} - x^{-1} + \frac{3}{2}x^{-2}$$

$$\frac{dy}{dx} = x^{-2} - 3x^{-3} = \frac{1}{x^2} - \frac{3}{x^3}$$

$$\text{(viii) } y = (x^2 - 2)(\sqrt{x}) = x^{\frac{5}{2}} - 2x^{\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}} - x^{\frac{1}{2}}$$

Edexcel AS Maths Differentiation 3 Exercise solutions

$$2. \text{ (i) } y = 2x - \frac{1}{x} = 2x - x^{-1}$$

$$\frac{dy}{dx} = 2 + x^{-2}$$

$$\text{At } (1, 1): \frac{dy}{dx} = 2 + (1)^{-2} = 3$$

$$\text{(ii) } y = 3 - \sqrt{x} = 3 - x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{1}{2}} = -\frac{1}{2\sqrt{x}}$$

$$\text{At } (4, 1): \frac{dy}{dx} = -\frac{1}{2\sqrt{4}} = -\frac{1}{4}$$

$$\text{(iii) } y = x^2\sqrt{x} = x^{\frac{5}{2}}$$

$$\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}}$$

$$\text{At } (1, 1): \frac{dy}{dx} = \frac{5}{2}(1)^{\frac{3}{2}} = \frac{5}{2}$$

$$3. y = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}} \Rightarrow \frac{dy}{dx} = -\frac{1}{2}x^{-\frac{3}{2}}$$

$$\text{When } x=1, y=1 \text{ and } \frac{dy}{dx} = -\frac{1}{2}(1)^{-\frac{3}{2}} = -\frac{1}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{1}{2}(x - 1)$$

$$y - 1 = -\frac{1}{2}x + \frac{1}{2}$$

$$y = -\frac{1}{2}x + \frac{3}{2}$$

$$4. y = \frac{1}{x} - \frac{2}{x^2} = x^{-1} - 2x^{-2} \Rightarrow \frac{dy}{dx} = -x^{-2} + 4x^{-3}$$

$$\text{When } x=2, y = \frac{1}{2} - \frac{1}{2} = 0, \frac{dy}{dx} = -(2)^{-2} + 4(2)^{-3} = 0.25$$

$$\text{As normal, } m = \frac{-1}{0.25} = -4$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -4(x - 2)$$

$$y = -4x + 8$$