Section 1: Introduction to differentiation

Solutions to Exercise level 2

1. $y = x^3 + 2x^2$

$\frac{dy}{dx} = 3x^2 + 4x$

When gradient is 4, $3x^2 + 4x = 4$

$3x^2 + 4x - 4 = 0$

$(3x-2)(x+2) = 0$

$x = \frac{2}{3}$ or $x = -2$

2. (i) When $x = 1$, $y = 2x^3 = 2 \times 1^3 = 2$

When $x = 1$, $y = 3x^2 - 1 = 3 \times 1^2 - 1 = 2$

so the point (1, 2) lies on both curves.

(ii) $y = 2x^3$

$\frac{dy}{dx} = 6x^2$

When $x = 1$, gradient $= 6 \times 1 = 6$

$y = 3x^2 - 1$

$\frac{dy}{dx} = 6x$

When $x = 1$, gradient $= 6 \times 1 = 6$

so the curves have the same gradient at this point.

(iii) The two curves touch each other at (1, 2).

3. $s = t^3 - 3t^2 - 9t$

$v = \frac{ds}{dt} = 3t^2 - 6t - 9$

When particle is stationary, $3t^2 - 6t - 9 = 0$

$t^2 - 2t - 3 = 0$

$(t-3)(t+1) = 0$

$t = 3$ or $t = -1$

Since time must be positive, $t = 3$.

The particle is stationary after 3 seconds.

4. (i) $y = x^3 - x^2 + x - 1$

$\Rightarrow \frac{dy}{dx} = 3x^2 - 2x + 1$

(ii) $y = (x^2 - 1)(x - 2)$

$= x^3 - 2x^2 - x + 2$

$\Rightarrow \frac{dy}{dx} = 3x^2 - 4x - 1$

5. $y = ax^3 + bx$

When $x = 1$, $y = a + b \quad \Rightarrow a + b = 8$

$\frac{dy}{dx} = 3ax^2 + b$

When $x = 1$, gradient $= 3a + b \quad \Rightarrow 3a + b = 12$

$3a + b = 12$

$a + b = 8$

Subtracting: $2a = 4$

$a = 2, b = 6$

6. $y = x^3 + x + 2$

$\frac{dy}{dx} = 3x^2 + 1$

When $x = 1$, $\frac{dy}{dx} = 3 \times 1^2 + 1 = 4$

When $x = 1$, $y = 1^3 + 1 + 2 = 4$

The tangent has gradient 4 and passes through the point (1, 4).

Equation of tangent is $y - 4 = 4(x - 1)$

$y - 4 = 4x - 4$

$y = 4x$

So the tangent passes through the origin.

Gradient of normal $= -\frac{1}{4}$

Equation of normal is $y - 4 = -\frac{1}{4}(x - 1)$

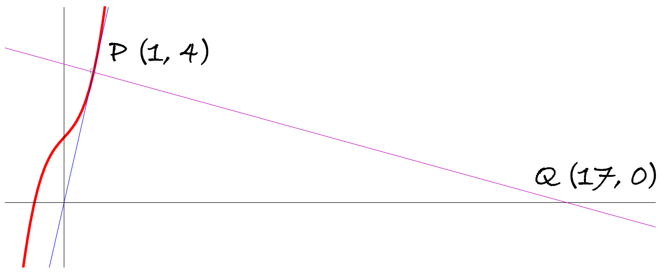
$4(y - 4) = -(x - 1)$

$4y - 16 = -x + 1$

$4y + x = 17$

When $y = 0$, $x = 17$, so Q is (17, 0).

Edexcel AS Maths Differentiation 1 Exercise solutions



Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 17 \times 4 = 34$

7. (i) $x = p \Rightarrow y = ap^2 + bp + c$

$\frac{dy}{dx} = 2ax + b$

so $x = p \Rightarrow \frac{dy}{dx} = 2ap + b$

Equation of tangent is $y - (ap^2 + bp + c) = (2ap + b)(x - p)$

$y = (2ap + b)x - ap^2 + c$

(ii) $y = 2apx - ap^2 + c$

(ii) If $b = 0$, the equation is $y = ax^2 + c$, so $(0, c)$ is the vertex of the graph. So for all values of a, the equation of the tangent at $x = 0$ is always $y = c$.

8. (i) $x = 0$, so curve (A) gives $y = 1$, and curve B gives $y = 1$, so the curves cross at $(0, 1)$.

(ii) For (A), $\frac{dy}{dx} = x^2 + 2$

so $x = 0 \Rightarrow$ gradient of curve $= 2$.

For (B), $\frac{dy}{dx} = 2x - \frac{1}{2}$,

so $x = 0 \Rightarrow$ gradient of curve $= -\frac{1}{2}$

(iii) The tangents of the two curves are perpendicular – they cross at right-angles.

(iv) For $y = ax^2 - \frac{1}{2}x + 1$, $\frac{dy}{dx} = 2ax - \frac{1}{2}$

so at $(0, 1)$, gradient of curve $= -\frac{1}{2}$ for any value of a.