Section 1: Introduction to vectors

Solutions to Exercise level 3

1. (i) $a\begin{pmatrix}2\\1\end{pmatrix}+b\begin{pmatrix}3\\2\end{pmatrix}=\begin{pmatrix}71\\33\end{pmatrix} \Rightarrow \begin{matrix}2a+3b=71\\a+2b=33\end{matrix}$

$$\begin{aligned}2a+3b&=71\\2a+4b&=66\\-b&=5\\b&=-5, a=43\end{aligned}$$

$$43\begin{pmatrix}2\\1\end{pmatrix}-5\begin{pmatrix}3\\2\end{pmatrix}=\begin{pmatrix}71\\33\end{pmatrix}$$

(ii) $2\begin{pmatrix}2\\1\end{pmatrix}-\begin{pmatrix}3\\2\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}$ and $2\begin{pmatrix}3\\2\end{pmatrix}-3\begin{pmatrix}2\\1\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix}$

Any point (p, q) can be reached by using $p\begin{pmatrix}1\\0\end{pmatrix}+q\begin{pmatrix}0\\1\end{pmatrix}$.

[Alternative approach: use simultaneous equations as above which gives $a=2p-3q$ and $b=2q-p$]

(iii) $a\begin{pmatrix}2\\1\end{pmatrix}+b\begin{pmatrix}3\\-2\end{pmatrix}=\begin{pmatrix}71\\33\end{pmatrix} \Rightarrow \begin{matrix}2a+3b=71\\a-2b=33\end{matrix}$

Solving these equations gives $a=\frac{241}{7}, b=\frac{5}{7}$ so not integers.

In general, $a\begin{pmatrix}2\\1\end{pmatrix}+b\begin{pmatrix}3\\-2\end{pmatrix}=\begin{pmatrix}p\\q\end{pmatrix} \Rightarrow \begin{matrix}2a+3b=p\\a-2b=q\end{matrix}$

$$\begin{aligned}2a+3b&=p\\2a-4b&=2q\\7b&=p-2q\\b&=\frac{p-2q}{7}\end{aligned}$$

So the points (p, q) must be such that $p-2q$ is a multiple of 7.