Edexcel AS Mathematics The binomial expansion

Section 1: Finding binomial expansions

Solutions to Exercise level 3 (Extension)

1. (i) $\left(1+\sqrt{3}\right)^4 = 1 + 4.\sqrt{3} + 6\left(\sqrt{3}\right)^2 + 4\left(\sqrt{3}\right)^3 + \left(\sqrt{3}\right)^4$

$= 1 + 4\sqrt{3} + 6(3) + 4(3)\sqrt{3} + 9$

$= 28 + 16\sqrt{3}$

(ii) $\left(1-\sqrt{3}\right)^4 = \left(1+\left(-\sqrt{3}\right)\right)^4$

$= 28 - 16\sqrt{3}$

2. (i) $M = x^3\rho$

(ii) $M = (x+\alpha)^3(\rho+\beta)$

$= (x^3 + 3\alpha x^2 +$ terms in α^2 or greater$)(\rho+\beta)$

$= (\rho+\beta)x^3 + 3\alpha\rho x^2 +$ terms in $\alpha^2, \alpha\beta$, or higher powers

so $M \approx (\rho+\beta)x^3 + 3\alpha\rho x^2$

(iii) If $x = 3$, $\rho = 10$ and $\alpha = 0.01$, $\beta = 0.02$,

additional mass $\approx \beta x^3 + 3\alpha\rho x^2$

$= (0.02)(27) + 3(0.01)(10)(9)$

$= 3.24$

Check: new mass $= (3.01)^3(10.02)$

$= 273.254........$

3. (i) $\left(1+\frac{1}{2}\right)^2 = 1 + 2\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 = 2\frac{1}{4} \quad (= 2.25)$

$\left(1+\frac{1}{3}\right)^3 = 1 + 3\left(\frac{1}{3}\right) + 3\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 = 2\frac{10}{27} \quad (\approx 2.037037....)$

$\left(1+\frac{1}{4}\right)^4 = 1 + 4\left(\frac{1}{4}\right) + 6\left(\frac{1}{4}\right)^2 + 4\left(\frac{1}{4}\right)^3 + \left(\frac{1}{4}\right)^4 = 2\frac{113}{256} \quad (= 2.44140625)$

(ii) $\left(1+\frac{1}{n}\right)^n \approx 1 + n\left(\frac{1}{n}\right) + \frac{n(n-1)}{1.2}\left(\frac{1}{n}\right)^2 + \frac{n(n-1)(n-2)}{1.2.3}\left(\frac{1}{n}\right)^3 + \frac{n(n-1)(n-2)(n-3)}{1.2.3.4}\left(\frac{1}{n}\right)^4$

$= 2 + \frac{n-1}{2n} + \frac{(n-1)(n-2)}{6n^2} + \frac{(n-1)(n-2)(n-3)}{24n^3}$

Edexcel AS Maths Binomial 1 Exercise solutions

(iii)

n	1st term	2nd term	3rd term	4th term	5th term	Approx.
1	1	1	0	0	0	2
2	1	1	0.25	0	0	2.25
3	1	1	0.333333	0.037037	0	2.37037
.....	.....	.....	.....	.....	.....	.....
10	1	1	0.45	0.12	0.021	2.591
100	1	1	0.495	0.1617	0.039212	2.695912
1000	1	1	0.4995	0.166167	0.041417	2.707084

As n gets bigger, all the 'ratios' of terms involving n in the approximation

$$\left(1+\frac{1}{n}\right)^n \approx 2+\frac{n-1}{2n}+\frac{(n-1)(n-2)}{6n^2}+\frac{(n-1)(n-2)(n-3)}{24n^3}$$

get closer and closer to 1, so in the limit the sequence of the Professor's calculations seems likely to tend to

$$\text{As } n \to \infty, \left(1+\frac{1}{n}\right)^n \to 2+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}$$

$$= 2\tfrac{17}{24} \qquad (\approx 2.7083333......)$$

(iv) Using a spreadsheet to calculate $\left(1+\frac{1}{n}\right)^n$ for large n leads to

n	$(1+(1/n))^n$
1	2
2	2.25
3	2.370370370
.....	.....
10	2.593742460
100	2.704813829
1000	2.716923932
10000	2.718145927
100000	2.718268237
1000000	2.718280469
10000000	2.718281694
100000000	2.718281786
1.00E+09	2.718282031
1.00E+10	2.718282053

You may recognise the limiting process as tending towards the constant e.