Edexcel AS Mathematics The binomial expansion

Section 1: Finding binomial expansions

Solutions to Exercise level 2

1. (i) Term in $x^4 = {}_{15}C_4(2x)^4 = 1365 \times 16x^4 = 21840x^4$

Coefficient of x^4 is 21840

(ii) Term in $x^{23} = {}_{25}C_{23}(3)^2(-x)^{23} = {}_{25}C_2(3)^2(-x)^{23}$

$= 300 \times 9 \times -x^{23} = -2700x^{23}$

Coefficient of x^{23} is -2700.

(iii) Term in $x^2 = {}_{10}C_4 x^6 \left(\frac{1}{x}\right)^4 = 210\frac{x^6}{x^4} = 210x^2$

Coefficient of x^2 is 210.

2. (i) zero $\left({}_{19}C_{10} = \frac{10}{10}({}_{19}C_9) = 92378 \right)$

(ii) 1 $\quad (= 10 - 9)$

(iii) $\frac{3}{7}$ $\quad \left({}_9C_6 = \frac{9\times8\times7\times6\times5\times4}{1\times2\times3\times4\times5\times6} \quad {}_9C_7 = \frac{9\times8\times7\times6\times5\times4\times3}{1\times2\times3\times4\times5\times6\times7} \right)$

3. $f(x) = \sin^4 x + 4\sin^3 x\cos x + 6\sin^2 x\cos^2 x + 4\sin x\cos^3 x + \cos^4 x$

$\Rightarrow f(45^\circ) = \left(\frac{1}{\sqrt{2}}\right)^4 + 4\left(\frac{1}{\sqrt{2}}\right)^3\left(\frac{1}{\sqrt{2}}\right) + 6\left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)^3 + \left(\frac{1}{\sqrt{2}}\right)^4$

$= 16\left(\frac{1}{\sqrt{2}}\right)^4$

$= 4$

4. $(1+x)^4 = 1 + 4x + 6x^2 + \ldots$

$(1-x)^7 = 1 - 7x + 21x^2 \ldots$

$(1+x)^4(1-x)^7 = \left(1 + 4x + 6x^2 + \ldots\right)\left(1 - 7x + 21x^2 + \ldots\right)$

$= (1 - 7x + 21x^2) + 4x(1 - 7x) + 6x^2 + \ldots$

$= 1 - 7x + 21x^2 + 4x - 28x^2 + 6x^2 + \ldots$

$= 1 - 3x - x^2 + \ldots$

5. (i) $(1-x)^{15} = 1 - 15x + \frac{15\times14}{1\times2}x^2 + \ldots = 1 - 15x + 105x^2 + \ldots$

Edexcel AS Maths Binomial 1 Exercise solutions

(ii) Putting $x = 0.01$: $(1-0.01)^{15} = 1 - 15\times 0.01 + 105\times 0.01^2 + \ldots$

$0.99^{15} = 1 - 0.15 + 0.0105 + \ldots = 0.8605$

(iii) Percentage error $= \dfrac{0.8605 - 0.99^{15}}{0.99^{15}} \times 100 = 0.051\%$

6. (i) $\left(1+\dfrac{x}{2}\right)^9 = 1 + 9\left(\dfrac{x}{2}\right) + \dfrac{9\times 8}{1\times 2}\left(\dfrac{x}{2}\right)^2 + \dfrac{9\times 8\times 7}{1\times 2\times 3}\left(\dfrac{x}{2}\right)^3 + \ldots$

$= 1 + \frac{9}{2}x + 9x^2 + \frac{21}{2}x^3 + \ldots$

(ii) Putting $x = 0.1$: $\left(1+\dfrac{0.1}{2}\right)^9 = 1 + \frac{9}{2}\times 0.1 + 9\times 0.1^2 + \frac{21}{2}\times 0.1^3 + \ldots$

$1.05^9 = 1 + 0.45 + 0.09 + 0.0105 + \ldots = 1.5505$

(iii) Percentage error $= \dfrac{1.5505 - 1.05^9}{1.05^9} \times 100 = -0.053\%$

7. $f(x) = (8 - 12x + 6x^2 - x^3)\,(1 - \frac{2}{x} + \frac{1}{x^2})$

$= 8 - 12x + 6x^2 - x^3 - \dfrac{16}{x} + 24 - 12x + 2x^2 + \dfrac{8}{x^2} - \dfrac{12}{x} + 6 - x$

$= \dfrac{8}{x^2} - \dfrac{28}{x} + 38 - 24x + 7x^2 - x^3$

Check: $f(1) = (2-1)^3(1-\frac{1}{1})^2 = 0$

and $f(1) = 8 - 28 + 38 - 24 + 7 - 1 = 0$ as expected.

$f(2) = (2-2)^3(1-\frac{1}{2})^2 = 0$

and $f(2) = 4 - 14 + 38 - 48 + 28 - 8 = 0$ as expected.

8. (i) $(1+x^2)^8 \approx 1 + 8x^2 + 28x^4 + 56x^6$

(ii) Put $x = 0.1$, $\Rightarrow (1+x^2)^8 = 1.01^8$

$1.01^8 \approx 1 + 8(0.01) + 28(0.0001) + 56(0.000001)$

$= 1.082856$

$= 1.083$ (to 3 dec. pl.)