

Section 2: Notation and proof

Exercise level 2 solutions

1. n is an even number so let $n = 2m$ where m is an integer

$$n^2 + n = (2m)^2 + 2m$$

$$= 4m^2 + 2m$$

$$= 2m(2m + 1)$$

Since $n^2 + n$ has a factor of 2, it is an even number.

2. (i) Let the first number be a , so the second number is $a + 1$, the third is $a + 2$, etc.

$$\text{The sum of the numbers} = a + (a + 1) + (a + 2) + (a + 3) + (a + 4)$$

$$= 5a + 10$$

$$= 5(a + 2)$$

Since 5 is a factor, the sum is a multiple of 5.

$$\begin{aligned} \text{(ii) Mean of numbers} &= \frac{\text{sum of numbers}}{5} \\ &= \frac{5(a + 2)}{5} \\ &= a + 2 \end{aligned}$$

$a + 2$ is the third number in the list, so the mean is equal to the third number.

3. (i) $6^2 - 4^2 = 36 - 16$

$$= 20$$

This is even, so the statement is disproved.

- (ii) 2 is a prime number and is not odd, so the statement is disproved.

- (iii) $2 \times 1 = 2$ has factors 1 and 2, so 2 factors = 2^1

$$3 \times 2 \times 1 = 6 \text{ has factors } 1, 2, 3, 6, \text{ so } 4 \text{ factors} = 2^2$$

$$4 \times 3 \times 2 \times 1 = 24 \text{ has factors } 1, 2, 3, 4, 6, 8, 12, 24 \text{ so } 8 \text{ factors} = 2^3$$

$$5 \times 4 \times 3 \times 2 \times 1 = 120 \text{ has factors } 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, \\ 30, 40, 60, 120, \text{ so } 16 \text{ factors} = 2^4$$

$$6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720 \text{ has factors } 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, \\ 18, 20, 24, 30, 36, 40, 45, 48, 60, 72, 80, 90, 120, 144, 180, 240, \\ 360, 720$$

so 30 factors $\neq 2^5$

so the statement is disproved

Edexcel AS Mathematics Problem solving

4. JANUARY	e.g.	JURY
FEBRUARY	e.g.	BEAR
MARCH	e.g.	CRAM
APRIL	e.g.	RAIL
AUGUST	e.g.	GUST
SEPTEMBER	e.g.	BEER
OCTOBER	e.g.	BOOT
NOVEMBER	e.g.	MORE
DECEMBER	e.g.	REED