

Section 2: Dividing and factorising polynomials

Solutions to Exercise level 3 (Extension)

$$1. \text{ (i) } f\left(-\frac{1}{2}\right) = 0 \Rightarrow -\frac{2}{8} - \frac{9}{4} - \frac{7}{2} + k = 0$$

$$\Rightarrow k = 6$$

$$\text{(ii) } 2x^3 - 9x^2 + 7x + 6 = 0$$

$$\Rightarrow (2x+1)(x^2 - 5x + 6) = 0$$

$$\Rightarrow (2x+1)(x-2)(x-3) = 0$$

$$\Rightarrow x = -\frac{1}{2}, 2, 3$$

$$2. \text{ (i) } f(x) = x^3 - (2a+1)x^2 + (a^2 - 2)x + a(a+2)$$

$$f(a) = a^3 - (2a+1)a^2 + (a^2 - 2)a + a(a+2)$$

$$= a^3 - 2a^3 - a^2 + a^3 - 2a + a^2 + 2a$$

$$= 0$$

$$f(a+2) = (a+2)^3 - (2a+1)(a+2)^2 + (a^2 - 2)(a+2) + a(a+2)$$

$$= (a^3 + 6a^2 + 12a + 8) + (-2a^3 - 8a^2 - 8a - a^2 - 4a - 4)$$

$$+ (a^3 + 2a^2 - 2a - 4) + (a^2 + 2a)$$

$$= 0$$

so $f(x) = (x+1)(x-a)(x-(a+2))$

$$\text{(ii) } a = \frac{2}{3} \Rightarrow y = f(x) = x^3 - \frac{7}{3}x^2 - \frac{14}{9}x + \frac{16}{9}$$

(or multiples of this, e.g. $y = 9x^3 - 21x^2 - 14x + 16$)

$$3. \text{ (i) } f(x) = (x-a)(x-3)^2$$

$$\text{(ii) } f(2) = (2-a)(-1)^2$$

$$= 2-a$$

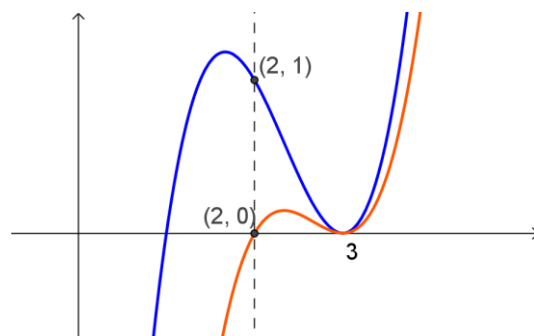
$$\Rightarrow 2-a = a^2 - 4a + 4$$

$$\Rightarrow a^2 - 3a + 2 = 0$$

$$\Rightarrow (a-1)(a-2) = 0$$

$$\Rightarrow a = 1, 2$$

When $x = 2$, $f(2) = (a-2)^2$
 so $a = 1 \Rightarrow f(2) = 1$
 and $a = 2 \Rightarrow f(2) = 0$
 so points are at $(2, 1)$ and $(2, 0)$



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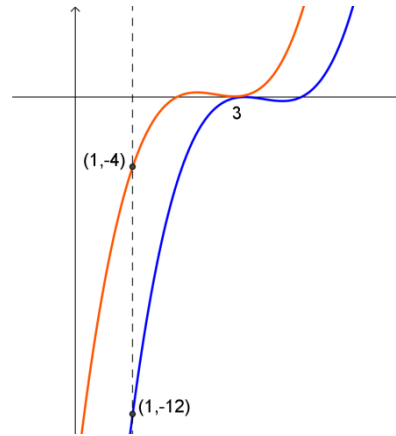
$$\begin{aligned} \text{(iii) } f(1) &= (1-a)(-2)^2 \\ &= 4-4a \\ \Rightarrow 4-4a &= a^2-10a+12 \\ \Rightarrow a^2-6a+8 &= 0 \\ \Rightarrow (a-2)(a-4) &= 0 \\ \Rightarrow a &= 2, 4 \end{aligned}$$

$$\text{When } x=1, f(1) = a^2 - 10a + 12$$

$$\text{so } a=2 \Rightarrow f(1) = -4$$

$$\text{and } a=4 \Rightarrow f(1) = -12$$

so points are at $(1, -4)$ and $(1, -12)$



$$\begin{aligned} \text{(iv) Both conditions } \Rightarrow a &= 2, \\ \text{so } y &= (x-2)(x-3)^2 \end{aligned}$$