Section 2: Dividing and factorising polynomials

Solutions to Exercise level 2

1. By inspection or long division:

$$6x^4 - 4x^3 + 3x^2 + 4x - 4 = (3x - 2)(2x^3 + x + 2)$$

so $6x^4 - 4x^3 + 3x^2 + 4x - 4 \div 3x - 2 = 2x^3 + x + 2$

2. (i) $f(2) = 16 + 4 - 2 - 18 = 0$

so $(x - 2)$ is a factor.

(ii) $2x^3 + x^2 - x - 18 = 0$

$$\Rightarrow (x - 2)(2x^2 + 5x + 9) = 0$$

For the quadratic factor, discriminant $= \sqrt{5^2 - 4(2)(9)} < 0$

so there are no further real roots other than $x = 2$.

3. $f(x) = 3x^3 + ax^2 + bx + 10$

$(x - 2)$ is a factor so $f(2) = 0$

$3 \times 2^3 + a \times 2^2 + b \times 2 + 10 = 0$

$24 + 4a + 2b + 10 = 0$

$4a + 2b = -34$

$2a + b = -17$

$(x + 1)$ is a factor so $f(-1) = 0$

$3(-1)^3 + a(-1)^2 + b(-1) + 10 = 0$

$-3 + a - b + 10 = 0$

$a - b = -7$

Adding: $3a = -24$

$a = -8, b = -1$

$3x^3 - 8x^2 - x + 10 = 0$

A quadratic factor is $(x - 2)(x + 1) = x^2 - x - 2$

By inspection or long division:

$(x^2 - x - 2)(3x - 5) = 0$

$(x - 2)(x + 1)(3x - 5) = 0$

The roots of the equation are $x = 2, -1, \frac{5}{3}$

4. (i) $f(x) = x^3 - x^2 - x - 2$

$f(2) = 2^3 - 2^2 - 2 - 2 = 8 - 4 - 2 - 2 = 0$

so by the factor theorem, $x - 2$ is a factor.

(ii) $x^3 - x^2 - x - 2 = (x - 2)(x^2 + x + 1)$
The quadratic expression $x^2 + x + 1$ cannot be factorised, so the expression has been factorised as far as possible.

(iii) The discriminant of $x^2 + x + 1$ is $1^2 - 4 \times 1 \times 1 = -3$, so the quadratic equation $x^2 + x + 1 = 0$ has no real roots.
Therefore the graph of $y = x^3 - x^2 - x - 2$ crosses the x-axis once only.

5. $3x^3 - 2x^2 - 11x + 10 = 0$
$f(x) = 3x^3 - 2x^2 - 11x + 10$
$f(1) = 3 - 2 - 11 + 10 = 0$ so $(x - 1)$ is a factor
$(x - 1)(3x^2 + x - 10) = 0$
$(x - 1)(3x - 5)(x + 2) = 0$
$x = 1$ or $x = \frac{5}{3}$ or $x = -2$

6. $2x^3 + 5x^2 - 14x - 8 = 0$
$f(x) = 2x^3 + 5x^2 - 14x - 8$
$f(1) = 2 + 5 - 14 - 8 = -15$
$f(-1) = -2 + 5 + 14 - 8 = 9$
$f(2) = 16 + 20 - 28 - 8 = 0$ so $(x - 2)$ is a factor
$(x - 2)(2x^2 + 9x + 4) = 0$
$(x - 2)(2x + 1)(x + 4) = 0$
$x = 2$ or $x = -\frac{1}{2}$ or $x = -4$

7. $4x^3 + 12x^2 - 7x - 30 = 0$
$f(x) = 4x^3 + 12x^2 - 7x - 30$
$f(1) = 4 + 12 - 7 - 30 = -21$
$f(-1) = -4 + 12 + 7 - 30 = -15$
$f(2) = 32 + 48 - 14 - 30 = 36$
$f(-2) = -32 + 48 + 14 - 30 = 0$ so $(x + 2)$ is a factor
$(x + 2)(4x^2 + 4x - 15) = 0$
$(x + 2)(2x + 5)(2x - 3) = 0$
$x = -2$ or $x = -\frac{5}{2}$ or $x = \frac{3}{2}$