Section 2: Dividing and factorising polynomials

Solutions to Exercise level 1

1. (i) By inspection or long division

$2x^3 - x^2 + 7x + 4 = (2x + 1)(x^2 - x + 4)$

so $(2x^3 - x^2 + 7x + 4) \div (2x + 1) = x^2 - x + 4$

(ii) By inspection or long division:

$x^3 + 2x^2 - 3 = (x - 1)(x^2 + 3x + 3)$

so $(x^3 + 2x^2 - 3) \div (x - 1) = x^2 + 3x + 3$

(iii) By inspection or long division:

$2x^2 + x - 1 = (2x - 1)(x + 1)$

so $(2x^2 + x - 1) \div (2x - 1) = x + 1$

2. (i) $f(x) = 2x^3 - 5x^2 - x + 6$

$f(-1) = 2(-1)^3 - 5(-1)^2 - (-1) + 6 = -2 - 5 + 1 + 6 = 0$

so by the factor theorem, $x + 1$ is a factor.

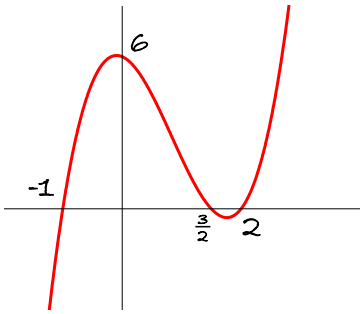
(ii) $2x^3 - 5x^2 - x + 6 = (x + 1)(2x^2 - 7x + 6)$

$= (x + 1)(2x - 3)(x - 2)$

(iii) $y = 2x^3 - 5x^2 - x + 6 = (x + 1)(2x - 3)(x - 2)$

When $x = 0$, $y = 6$

When $y = 0$, $x = -1$ or $x = \frac{3}{2}$ or $x = 2$



3. (i) $f(x) = x^3 + ax^2 - 4x + 12$

$f(2) = 2^3 + a \times 2^2 - 4 \times 2 + 12$

$= 8 + 4a - 8 + 12$

$= 4a + 12$

$x - 2$ is a factor so by the factor theorem $f(2) = 0$

$4a + 12 = 0$

$a = -3$

(ii) $x^3 - 3x^2 - 4x + 12 = (x + 2)(ax^2 + bx + c)$

By inspection $x^3 - 3x^2 - 4x + 12 = (x - 2)(x^2 - x - 6)$

$= (x - 2)(x + 2)(x - 3)$

4. $f(1) = 1 + 1 + b - 3 + 3 = 0$

$\Rightarrow b = -2$