Section 2: Circles

Solutions to Exercise level 2

1. (i) $x^2 + y^2 = 8$

Substituting in $y = 4 - x$ gives $x^2 + (4-x)^2 = 8$

$$x^2 + 16 - 8x + x^2 = 8$$
$$2x^2 - 8x + 8 = 0$$
$$x^2 - 4x + 4 = 0$$
$$(x-2)^2 = 0$$

The line meets the circle at just one point, so the line touches the circle and is therefore a tangent.

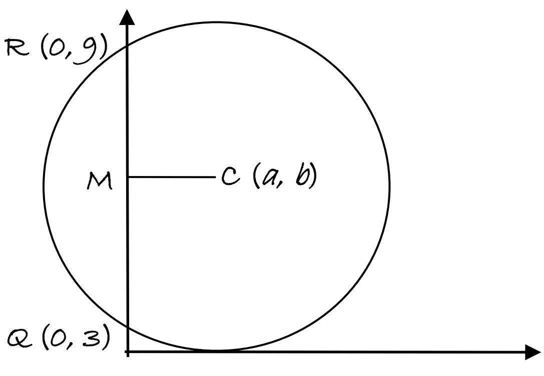
(ii) $x^2 + y^2 = 25$

Substituting in $4y = 3x - 25 \Rightarrow y = \frac{3x-25}{4}$

gives
$$x^2 + y^2 = 25$$
$$x^2 + \left(\frac{3x-25}{4}\right)^2 = 25$$
$$x^2 + \frac{(3x-25)^2}{16} = 25$$
$$16x^2 + 9x^2 - 150x + 625 = 400$$
$$25x^2 - 150x + 225 = 0$$
$$x^2 - 6x + 9 = 0$$
$$(x-3)^2 = 0$$

The line meets the circle at just one point, so the line touches the circle and is therefore a tangent.

2.



The midpoint M of QR is (0, 6).

Since a diameter which passes through M is perpendicular to QR, then the line CM must be horizontal, and therefore $b = 6$.
Since the circle touches the x-axis, the radius of the circle must be b, i.e. 6.
The equation of the circle is therefore $(x-a)^2+(y-6)^2=6^2$
The circle passes through (0, 3), so $(0-a)^2+(3-6)^2=6^2$

$$a^2+9=36$$
$$a^2=27$$
$$a=\pm\sqrt{27}=\pm3\sqrt{3}$$

The equation of the circle is either $(x-3\sqrt{3})^2+(y-6)^2=36$
or $(x+3\sqrt{3})^2+(y-6)^2=36$.

3. $x^2+y^2=65$
$2y+x=10 \quad \Rightarrow x=10-2y$
Substituting in: $(10-2y)^2+y^2=65$

$$100-40y+4y^2+y^2=65$$
$$5y^2-40y+35=0$$
$$y^2-8y+7=0$$
$$(y-1)(y-7)=0$$
$$y=1 \text{ or } y=7$$

When $y = 1$, $x=10-2\times1=8$
When $y = 7$, $x=10-2\times7=-4$
so P is (8, 1) and Q is (-4, 7)
Length $PQ=\sqrt{(8-(-4))^2+(1-7)^2}=\sqrt{144+36}=\sqrt{180}$

4. Substituting $y=x+1$ into $(x-1)^2+(y+2)^2=k$:

$$(x-1)^2+(x+1+2)^2=k$$
$$(x-1)^2+(x+3)^2=k$$
$$x^2-2x+1+x^2+6x+9=k$$
$$2x^2+4x+10-k=0$$

If there are no intersections, then $b^2-4ac<0$

$$a=2, b=4, c=10-k$$
$$4^2-4\times2(10-k)<0$$
$$16-8(10-k)<0$$
$$2-(10-k)<0$$
$$2-10+k<0$$
$$k<8$$

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5. (i) Gradient of PR $= \dfrac{7-6}{5-(-2)} = \dfrac{1}{7}$

Gradient of QR $= \dfrac{7-0}{5-6} = \dfrac{7}{-1} = -7$

Gradient of PR × gradient of QR $= \dfrac{1}{7} \times -7 = -1$

so PR and QR are perpendicular.

(ii) The angle in a semicircle is 90°, so PQ must be a diameter.

(iii) Since PQ is a diameter, the centre C of the circle is the midpoint of PQ.

$$C = \left(\frac{-2+6}{2}, \frac{6+0}{2}\right) = (2,3)$$

Radius of circle = length CQ $= \sqrt{(6-2)^2 + (0-3)^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$

Equation of circle is $(x-2)^2 + (y-3)^2 = 25$.

6. (i) $x^2 + y^2 = 17$

(ii) Substituting x = -4 and y = -1: $x^2 + y^2 = (-4)^2 + (-1)^2 = 16 + 1 = 17$

(iii) Gradient of radius OP $= \dfrac{-1-0}{-4-0} = \dfrac{1}{4}$

Tangent to circle at P is perpendicular to radius OP

so gradient of tangent = -4

Equation of tangent is $y - (-1) = -4(x - (-4))$

$$y + 1 = -4(x+4)$$

$$y + 1 = -4x - 16$$

$$y + 4x + 17 = 0$$

(iv) $x + y = 3 \quad \Rightarrow y = 3 - x$

Substituting into equation of circle:

$$x^2 + (3-x)^2 = 17$$

$$x^2 + 9 - 6x + x^2 = 17$$

$$2x^2 - 6x - 8 = 0$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$x = 4 \text{ or } x = -1$$

When x = 4, $y = 3 - 4 = -1$

When x = -1, $y = 3 - (-1) = 4$

Coordinates of Q and R are (4, -1) and (-1, 4).

(v) Tangent is $y+4x+17=0$

Substituting in $y=3-x$ gives $(3-x)+4x+17=0$

$$20+3x=0$$

$$x=-\frac{20}{3}$$

When $x=-\frac{20}{3}$, $y=3+\frac{20}{3}=\frac{29}{3}$

Coordinates of S are $\left(-\frac{20}{3},\frac{29}{3}\right)$

7. $\left.\begin{array}{l} y=x^2+8 \\ y=2x^2+x+6 \end{array}\right\}$

$\Rightarrow x^2+8=2x^2+x+6$

$\Rightarrow \quad x^2+x-2=0$

$\Rightarrow (x-1)(x+2)=0$

$\Rightarrow x=1, y=9$ or $x=-2, y=12$

So length PQ $=\sqrt{3^2+3^2}$

$=3\sqrt{2}$