Section 1: Points and straight lines

Solutions to Exercise level 3 (Extension)

1. (i) Midpoint of EF $= \left(\frac{2+4}{2}, \frac{5+1}{2}\right) = (3, 3)$

Midpoint of FG $= \left(\frac{4+(-2)}{2}, \frac{1+(-3)}{2}\right) = (1, -1)$

Midpoint of EG $= \left(\frac{2+(-2)}{2}, \frac{5+(-3)}{2}\right) = (0, 1)$

Median from midpoint of EF (3, 3) to G (-2, -3)

Gradient of median $= \frac{-3-3}{-2-3} = \frac{-6}{-5} = \frac{6}{5}$

Equation of median is $y - 3 = \frac{6}{5}(x-3)$

$5(y-3) = 6(x-3)$

$5y - 15 = 6x - 18$

$5y = 6x - 3$

Median from midpoint of FG (1, -1) to E (2, 5)

Gradient of median $= \frac{5-(-1)}{2-1} = \frac{6}{1} = 6$

Equation of median is $y - (-1) = 6(x-1)$

$y + 1 = 6x - 6$

$y = 6x - 7$

Median from midpoint of EG (0, 1) to F (4, 1)

Gradient of median $= \frac{1-1}{4-0} = \frac{0}{4} = 0$

Equation of median is $y = 1$

(ii) Equation of first median is $5y = 6x - 3$

Substituting $x = \frac{4}{3}$ gives $5y = 6 \times \frac{4}{3} - 3 = 8 - 3 = 5$

$y = 1$

so $\left(\frac{4}{3}, 1\right)$ lies on the median.

Equation of second median is $y = 6x - 7$

Substituting $x = \frac{4}{3}$ gives $y = 6 \times \frac{4}{3} - 7 = 8 - 7 = 1$

so $\left(\frac{4}{3}, 1\right)$ lies on the median.

Equation of third median is $y = 1$, so $\left(\frac{4}{3}, 1\right)$ lies on the median.

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2. (i) Let the triangle be ABC.

Let A be the intersection point of $y+3x=11$ and $3y=x+3$.

$y+3x=11 \quad \Rightarrow y=11-3x$

Substituting into $3y=x+3$ gives $3(11-3x)=x+3$

$33-9x=x+3$

$30=10x$

$x=3$

When $x=3$, $y=11-3\times3=2$

The coordinates of A are (3, 2).

Let B be the intersection point of $3y=x+3$ and $7y+x=37$

$3y=x+3 \quad \Rightarrow x=3y-3$

Substituting into $7y+x=37$ gives $7y+3y-3=37$

$10y=40$

$y=4$

When $y=4$, $x=3\times4-3=9$

The coordinates of B are (9, 4).

Let C be the intersection point of $7y+x=37$ and $y+3x=11$

$y+3x=11 \quad \Rightarrow y=11-3x$

Substituting into $7y+x=37$ gives $7(11-3x)+x=37$

$77-21x+x=37$

$40=20x$

$x=2$

When $x=2$, $y=11-3\times2=5$

The coordinates of C are (2, 5).

The vertices of the triangle are (3, 2), (9, 4) and (2, 5).

(ii) AB is the line $y+3x=11 \quad \Rightarrow y=11-3x$

so the gradient of AB is -3.

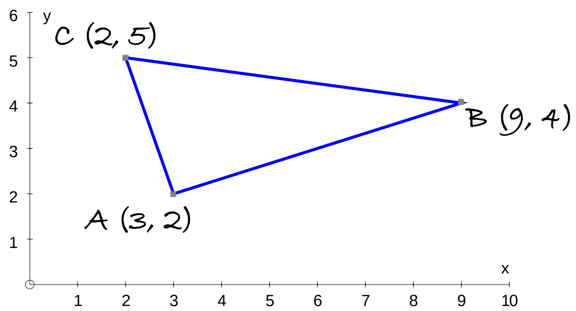
BC is the line $3y=x+3 \quad \Rightarrow y=\frac{1}{3}x+1$

so the gradient of BC is $\frac{1}{3}$.

Gradient of AB × gradient of AC $=-3\times\frac{1}{3}=-1$

so AB and AC are perpendicular, and therefore the triangle is right-angled.

(iii)



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$$AB = \sqrt{(3-9)^2 + (2-4)^2} = \sqrt{36+4} = \sqrt{40}$$

$$AC = \sqrt{(3-2)^2 + (2-5)^2} = \sqrt{1+9} = \sqrt{10}$$

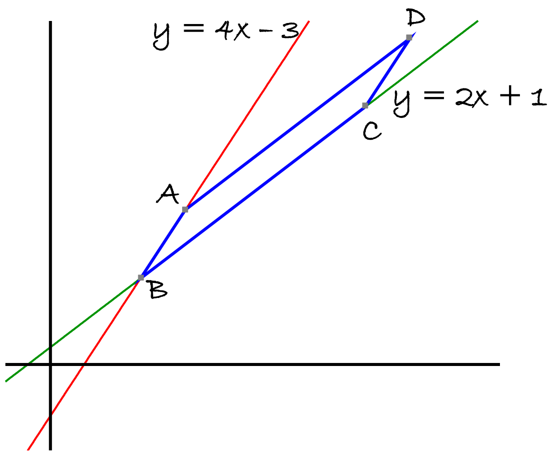
$$\begin{aligned}\text{Area of triangle} &= \tfrac{1}{2} \times AB \times AC \\ &= \tfrac{1}{2}\sqrt{40}\sqrt{10} \\ &= \tfrac{1}{2}\sqrt{4}\sqrt{10}\sqrt{10} \\ &= \tfrac{1}{2} \times 2 \times 10 \\ &= 10\end{aligned}$$

3. (i) B is the intersection point of $y = 4x - 3$ and $y = 2x + 1$.

$$4x - 3 = 2x + 1$$
$$2x = 4$$
$$x = 2$$

When $x = 2$, $y = 4 \times 2 - 3 = 5$

The coordinates of B are $(2, 5)$.



(ii) AD is parallel to BC, so AD has gradient 2.

AD passes through the point $(3, 9)$.

Equation of AD is $y - 9 = 2(x - 3)$

$$y - 9 = 2x - 6$$
$$y = 2x + 3$$

(iii) CD is parallel to AB, so CD has gradient 4.

CD passes through the point $(7, 15)$.

Equation of CD is $y - 15 = 4(x - 7)$

$$y - 15 = 4x - 28$$
$$y = 4x - 13$$

(iv) D is the intersection point of AD and CD.

$$2x + 3 = 4x - 13$$
$$16 = 2x$$
$$x = 8$$

When $x = 8$, $y = 2 \times 8 + 3 = 19$

The coordinates of D are $(8, 19)$.

4. Midpoint of AB $= \left(\dfrac{4+10}{2}, \dfrac{2+12}{2}\right) = (7, 7)$

Gradient of AB $= \dfrac{10}{6} = \dfrac{5}{3}$

Gradient of perpendicular to AB $= -\dfrac{3}{5}$

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Equation of perpendicular bisector is

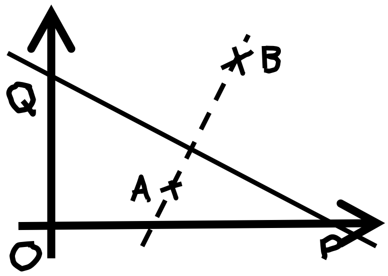
$$y-7=-\tfrac{3}{5}(x-7)$$

$$\Rightarrow 5y+3x=56$$

so $P=(\tfrac{56}{3},0)$ and $Q\ (0,\tfrac{56}{5})$

Area of $OPQ=\tfrac{1}{2}(\tfrac{56}{3})(\tfrac{56}{5})$

$$=104\tfrac{8}{15}\quad (\approx 104.53)$$



5. gradient $AB=\tfrac{3}{5}$

Equation of first line is $y-4=-\tfrac{5}{3}(x-8)$

$$\Rightarrow 3y=-5x+52$$

so $P=(\tfrac{52}{5},0)$ and $Q=(0,\tfrac{52}{3})$

Equation of second line is $y-1=-\tfrac{5}{3}(x-3)$

$$\Rightarrow 3y=-5x+18$$

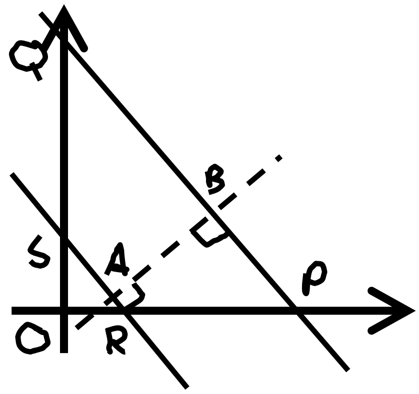
so $R=(\tfrac{18}{5},0)$ and $S=(0,6)$

So area PQSR = area of OPQ – area of ORS

$$=\tfrac{1}{2}(\tfrac{52}{5})(\tfrac{52}{3})-\tfrac{1}{2}(\tfrac{18}{5})(6)$$

$$=79\tfrac{1}{3}\quad (\approx \tfrac{238}{3})$$

The shape is a trapezium (since PQ and RS are parallel)



6. gradient $AB=-\tfrac{3}{4}$

$\Rightarrow$ eqtn of AB is $y-2=-\tfrac{3}{4}(x-5)$

$$\Rightarrow 4y=-3x+23 \quad (1)$$

so gradient $CD=\tfrac{4}{3}$

$\Rightarrow$ eqtn of CD is $y-6=\tfrac{4}{3}(x-6)$

$$\Rightarrow 3y=4x-6 \quad (2)$$

$(1)\times 4\ \Rightarrow 12y=16x-24$

$(2)\times 3 \Rightarrow 12y=-9x+69$

subtracting $\Rightarrow 0=25x-93$

$$\Rightarrow x=\tfrac{93}{25},\ y=\tfrac{74}{25}\ \text{ at point D}$$

7. (i) Midpoint of $AB=\left(\dfrac{4+2}{2},\dfrac{5+1}{2}\right)=(3,3)$

Midpoint of $CD=\left(\dfrac{7+-1}{2},\dfrac{1+5}{2}\right)=(3,3)$

(ii) Gradient of $AB=\dfrac{5-1}{4-2}=2$

Gradient of CD $= \dfrac{1-5}{7-(-1)} = -\dfrac{1}{2}$

(iii) AB and CD cross at right-angles at their midpoints, so ACBD is a rhombus.

(iv) Length AB $= \sqrt{(4-2)^2 + (5-1)^2} = \sqrt{4+16} = \sqrt{20}$

Length CD $= \sqrt{(7-(-1))^2 + (1-5)^2} = \sqrt{64+16} = \sqrt{80}$

Area $= \frac{1}{2}\sqrt{20}\sqrt{80} = 20$