Section 1: Points and straight lines

Solutions to Exercise level 2

1. (i) Gradient of AB $= \frac{y_1 - y_2}{x_1 - x_2} = \frac{1-y}{3-6} = \frac{1-y}{-3}$

Gradient of AB $= 2 \Rightarrow \frac{1-y}{-3} = 2$

$\Rightarrow 1 - y = -6$

$y = 7$

(ii) Distance AB is 5

$\sqrt{(3-6)^2 + (1-y)^2} = 5$

$9 + (1-y)^2 = 25$

$(1-y)^2 = 16$

$1 - y = \pm 4$

$y = 1 - 4$ or $1 + 4$

$y = -3$ or 5

(iii) If A, B and C are collinear, gradient of AB = gradient of AC.

Gradient of AC $= \frac{y_1 - y_2}{x_1 - x_2} = \frac{1-(-2)}{3-12} = \frac{3}{-9} = -\frac{1}{3}$

From (i), gradient of AB $= \frac{1-y}{-3}$

$\frac{1-y}{-3} = -\frac{1}{3}$

$1 - y = 1$

$y = 0$

(iv) If AB is perpendicular to BC, then grad AB × grad BC = -1

From (i), gradient of AB $= \frac{1-y}{-3}$

Gradient of BC $= \frac{y_1 - y_2}{x_1 - x_2} = \frac{y-(-2)}{6-12} = \frac{y+2}{-6}$

$$\frac{1-y}{-3}\times\frac{y+2}{-6}=-1$$
$$(1-y)(y+2)=-18$$
$$2-y-y^2=-18$$
$$y^2+y-20=0$$
$$(y+5)(y-4)=0$$
$$y=-5 \text{ or } y=4$$

(v) Length AB = length BC
$$\sqrt{(3-6)^2+(1-y)^2}=\sqrt{(6-12)^2+(y-(-2))^2}$$
$$9+(1-y)^2=36+(y+2)^2$$
$$1-2y+y^2=27+y^2+4y+4$$
$$0=6y+30$$
$$y=-5$$

2. (i) Gradient of $y=4x-1$ is 4
Gradient of parallel line = 4
Equation of line is $y-3=4(x-2)$
$$y-3=4x-8$$
$$y=4x-5$$

(ii) Gradient of $y=2x+7$ is 2
Gradient of perpendicular line is $-\frac{1}{2}$
Equation of line is $y-2=-\frac{1}{2}(x-1)$
$$2(y-2)=-(x-1)$$
$$2y-4=-x+1$$
$$2y+x=5$$

(iii) $3y+x=10\Rightarrow y=-\frac{1}{3}x+\frac{10}{3}$
Gradient is $-\frac{1}{3}$
Gradient of parallel line is $-\frac{1}{3}$
Equation of line is $y-(-1)=-\frac{1}{3}(x-4)$
$$3(y+1)=-(x-4)$$
$$3y+3=-x+4$$
$$3y+x=1$$

(iv) $3x+4y=12\Rightarrow y=-\frac{3}{4}x+3$
Gradient is $-\frac{3}{4}$
Gradient of perpendicular line is $\frac{4}{3}$

Equation of line is $y-0=\frac{4}{3}(x-(-3))$

$3y=4(x+3)$

$3y=4x+12$

(v) $x+5y+8=0 \Rightarrow y=-\frac{1}{5}x-\frac{8}{5}$

Gradient is $-\frac{1}{5}$

Gradient of parallel line is $-\frac{1}{5}$

Equation of line is $y-(-6)=-\frac{1}{5}(x-(-1))$

$5(y+6)=-(x+1)$

$5y+30=-x-1$

$5y+x+31=0$

3. (i) Gradient of AB $=\frac{y_1-y_2}{x_1-x_2}=\frac{6-2}{1-3}=\frac{4}{-2}=-2$

Equation of AB is $y-6=-2(x-1)$

$y-6=-2x+2$

$y+2x=8$

(ii) Gradient of AB $=\frac{y_1-y_2}{x_1-x_2}=\frac{-1-3}{8-(-2)}=\frac{-4}{10}=-\frac{2}{5}$

Equation of AB is $y-(-1)=-\frac{2}{5}(x-8)$

$5(y+1)=-2(x-8)$

$5y+5=-2x+16$

$5y+2x=11$

(iii) Gradient of AB $=\frac{y_1-y_2}{x_1-x_2}=\frac{2-(-4)}{-5-7}=\frac{6}{-12}=-\frac{1}{2}$

Equation of AB is $y-2=-\frac{1}{2}(x-(-5))$

$2(y-2)=-(x+5)$

$2y-4=-x-5$

$2y+x+1=0$

(iv) Gradient of AB $=\frac{y_1-y_2}{x_1-x_2}=\frac{-5-1}{-3-5}=\frac{-6}{-8}=\frac{3}{4}$

Equation of AB is $y-(-5)=\frac{3}{4}(x-(-3))$

$4(y+5)=3(x+3)$

$4y+20=3x+9$

$4y=3x-11$

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4. Gradient of EF $= \dfrac{3-(-1)}{1-2} = \dfrac{4}{-1} = -4$

Gradient of FG $= \dfrac{5-3}{3-1} = \dfrac{2}{2} = 1$

Gradient of GH $= \dfrac{1-5}{4-3} = \dfrac{-4}{1} = -4$

Gradient of EH $= \dfrac{1-(-1)}{4-2} = \dfrac{2}{2} = 1$

EF is parallel to GH and FG is parallel to EH
so EFGH is a parallelogram.

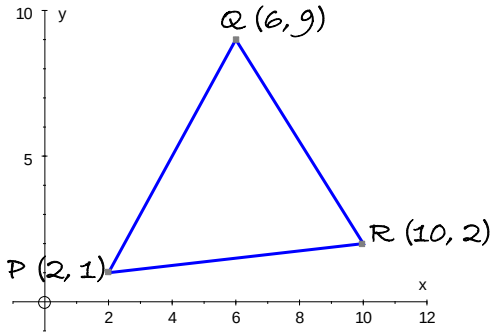
If EFGH were a rhombus, all the sides would be equal.

$EF^2 = (2-1)^2 + (-1-3)^2 = 1^2 + (-4)^2 = 17$

$FG^2 = (1-3)^2 + (3-5)^2 = (-2)^2 + (-2)^2 = 8$

The lengths of EF and FG are not equal, so EFGH is not a rhombus.

5. (i)



(ii) $PQ = \sqrt{(6-2)^2 + (9-1)^2} = \sqrt{16+64} = \sqrt{80}$

$PR = \sqrt{(10-2)^2 + (2-1)^2} = \sqrt{64+1} = \sqrt{65}$

$QR = \sqrt{(10-6)^2 + (2-9)^2} = \sqrt{16+49} = \sqrt{65}$

Since PR and QR are the same length, the triangle is isosceles.

(iii) Take the base of the triangle as PQ.
Let M be the midpoint of PQ

$M = \left(\dfrac{2+6}{2}, \dfrac{1+9}{2}\right) = (4, 5)$

Height of triangle is $MR = \sqrt{(10-4)^2 + (2-5)^2} = \sqrt{36+9} = \sqrt{45}$

$$\text{Area of triangle} = \tfrac{1}{2} \times PQ \times MR$$
$$= \tfrac{1}{2}\sqrt{80}\sqrt{45}$$
$$= \tfrac{1}{2}\sqrt{16 \times 5}\sqrt{9 \times 5}$$
$$= \tfrac{1}{2} \times 4\sqrt{5} \times 3\sqrt{5}$$
$$= 6 \times 5$$
$$= 30$$

6. (i) $\text{gradient BA} = \dfrac{5-0}{(-1)-1} = -\dfrac{5}{2}$

(ii) $\text{gradient BC} = \dfrac{4-0}{11-1} = \dfrac{2}{5}$

$\text{gradient BA} \times \text{gradient BC} = -\dfrac{5}{2} \times \dfrac{2}{5} = -1$, so BA and BC are perpendicular to each other.

(iii) $y - 4 = -\tfrac{5}{2}(x - 11)$

$\Rightarrow 2y + 5x = 63$

(iv) $y - 5 = \tfrac{2}{5}(x + 1)$

$\Rightarrow 5y - 2x = 27$

(v) $2y + 5x = 63 \quad (1)$
$5y - 2x = 27 \quad (2)$

mult (1) x 5 $\quad 10y + 25x = 315$

mult (2) x 2 $\quad 10y - 4x = 54$

subtracting $\quad 29x = 261$

$\Rightarrow x = 9, y = 9$

so D is the point $(9, 9)$