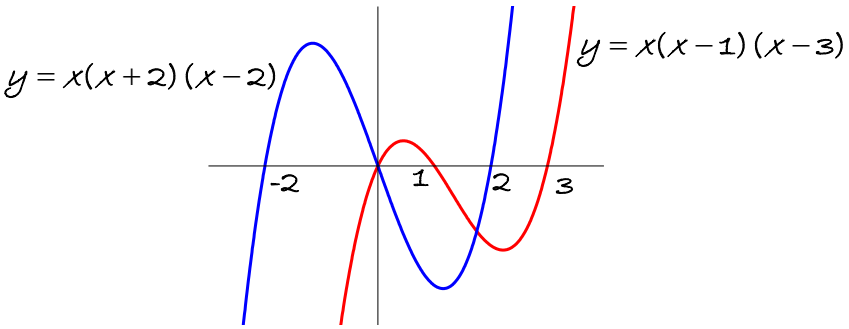
Edexcel AS Mathematics Graphs and transformations

Section 1: Sketching graphs of functions

Solutions to Exercise level 2

1. (i)



(ii) $x(x-1)(x-3) = x(x+2)(x-2)$

$x(x^2 - 4x + 3) = x(x^2 - 4)$

$x^3 - 4x^2 + 3x = x^3 - 4x$

$-4x^2 + 7x = 0$

$4x^2 - 7x = 0$

$x(4x - 7) = 0$

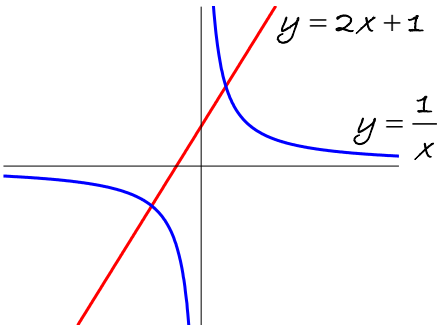
$x = 0$ or $x = \frac{7}{4}$

When $x = 0$, $y = 0$

When $x = \frac{7}{4}$, $y = \frac{7}{4}(\frac{7}{4} - 1)(\frac{7}{4} - 3) = \frac{7}{4} \times \frac{3}{4} \times -\frac{5}{4} = -\frac{105}{64}$

The coordinates of the points of intersection are $(0, 0)$ and $\left(\frac{7}{4}, -\frac{105}{64}\right)$.

2. (i)



(ii) $\frac{1}{x} = 2x + 1$

$1 = x(2x + 1)$

$1 = 2x^2 + x$

$2x^2 + x - 1 = 0$

$(2x - 1)(x + 1) = 0$

$x = \frac{1}{2}$ or $x = -1$

When $x = \frac{1}{2}$, $y = 2$

When $x = -1$, $y = -1$

The points of intersection are $(\frac{1}{2}, 2)$ and (-1, -1).

3. (i) $\frac{4}{x} = 5 - x$

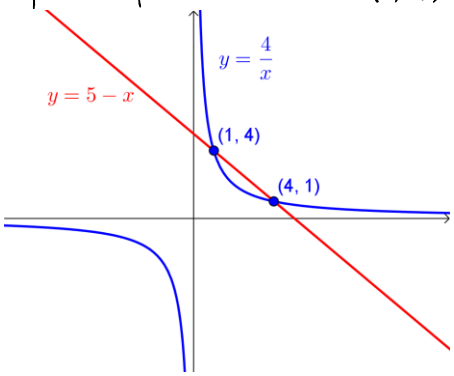
$4 = 5x - x^2$

$x^2 - 5x + 4 = 0$

$(x - 1)(x - 4) = 0$

When $x = 1$, $y = 4$ and when $x = 4$, $y = 1$

so the points of intersection are (1, 4) and (4, 1).



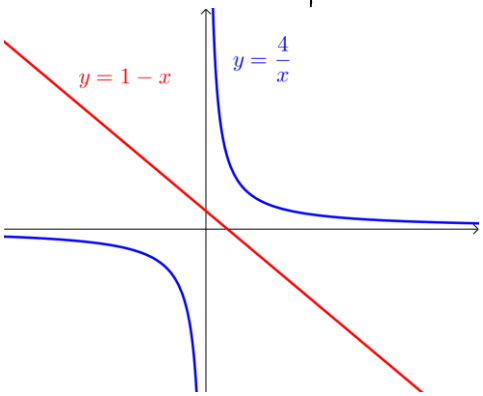
(ii) $\frac{4}{x} = 1 - x$

$4 = x - x^2$

$x^2 - x + 4 = 0$

Discriminant $= (-1)^2 - 4 \times 1 \times 4 = 1 - 16 < 0$

so there are no intersection points.



(iii) $\frac{4}{x} = k - x$

$4 = kx - x^2$

$x^2 - kx + 4 = 0$

If there are no intersections, discriminant < 0

$(-k)^2 - 4 \times 1 \times 4 < 0$

$k^2 - 16 < 0$

$k^2 < 16$

$-4 < k < 4$

4. Let F be the force between the particles, and let d be the distance between them.

$F = \frac{k}{d^2}$

When $d = 1, F = 90 \Rightarrow 90 = \frac{k}{1}$

so $F = \frac{90}{d^2}$

(i) When $d = 5$, $F = \frac{90}{5^2} = 3.6$

so the force is 3.6 N.

(ii) When $F = 2$, $2 = \frac{90}{d^2} \Rightarrow d^2 = 45 \Rightarrow d = 6.71$

The distance between them must be at least 6.71 cm (3 s.f.)

5. Let v be the speed, and let d be the distance travelled.

$v = k\sqrt{d}$

When $d = 4, v = 10 \Rightarrow 10 = k\sqrt{4} \Rightarrow k = 5$

$v = 5\sqrt{d}$

(i) When $d = 30$, $v = 5\sqrt{30} = 27.4$

so the speed is 27.4 ms^{-1} (3 s.f.)

(ii) When $v = 50$, $50 = 5\sqrt{d} \Rightarrow \sqrt{d} = 10 \Rightarrow d = 100$

so it has travelled 100 m.