Edexcel AS Mathematics Equations and inequalities

Section 2: Inequalities

Solutions to Exercise level 3 (Extension)

1. If John's age is x years, and his mother's age is y years, then

$$x < \tfrac{1}{2}y \qquad (1)$$
$$x + y > 60 \qquad (2)$$
$$y = 26 + x \qquad (3)$$

Substituting (3) into (1) $\Rightarrow 2x < 26 + x$

$$\Rightarrow \quad x < 26$$

Substituting (3) into (2) $\Rightarrow x + (26 + x) > 60$

$$\Rightarrow \quad 2x > 34$$
$$\Rightarrow \quad x > 17$$

so John's age is between 18 and 25 inclusive.

2. Area: $\quad x(x-3) \le 88$

Perimeter: $2x + 2(x-3) \ge 30$

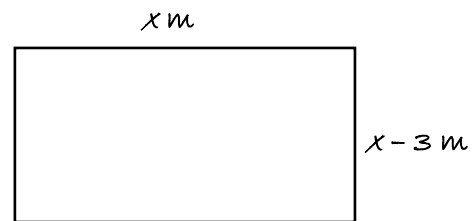$$\Rightarrow \quad \begin{array}{l} x^2 - 3x - 88 \le 0 \quad (1) \\ 4x - 36 \ge 0 \quad (2) \end{array}$$

(1) $\Rightarrow (x-11)(x+8) \le 0$

$$\Rightarrow \quad x \le 11 \quad \text{(precisely: } -8 \le x \le 11\text{)}$$

(2) $\Rightarrow x \ge 9$

so the length of the room is between 9 and 11 metres.

x m

$x - 3$ m

3. Volume of cone $= \tfrac{1}{3}\pi r^2 h$

$$\Rightarrow \tfrac{1}{3}\pi r^2 h \le 25 \quad (1)$$

For the slant height, $l < 2r$

and $l^2 = r^2 + h^2$

$$\Rightarrow r^2 + h^2 < 4r^2$$
$$\Rightarrow r^2 > \tfrac{1}{3}h^2$$

So (1) $\Rightarrow \tfrac{1}{9}\pi h^3 < 25$

$$\Rightarrow \quad h^3 < \tfrac{225}{\pi}$$
$$\Rightarrow \quad h < 4.15 \text{ m} \quad (3 \text{ s.f.})$$

l

h

r