Section 1: Simultaneous equations

Solutions to Exercise level 2

1. (i) $7x^2 + y^2 = 64$ (1)

$x + y = 4$ (2)

(2) $\Rightarrow y = 4 - x$

Substituting into (1): $7x^2 + (4-x)^2 = 64$

$7x^2 + 16 - 8x + x^2 = 64$

$8x^2 - 8x - 48 = 0$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3$ or $x = -2$

When $x = 3$, $y = 4 - 3 = 1$

When $x = -2$, $y = 4 - (-2) = 6$

The solutions are $x = 3$, $y = 1$ and $x = -2$, $y = 6$

Check: $x = 3, y = 1 \Rightarrow 7x^2 + y^2 = 63 + 1 = 64$

$x = -2, y = 6 \Rightarrow 7x^2 + y^2 = 28 + 36 = 64$

(ii) $3x^2 - 2y^2 = -5$ (1)

$y - x = 1$ (2)

(2) $\Rightarrow y = 1 + x$

Substituting into (1): $3x^2 - 2(1+x)^2 = -5$

$3x^2 - 2(1 + 2x + x^2) = -5$

$3x^2 - 2 - 4x - 2x^2 = -5$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x = 1$ or $x = 3$

When $x = 1$, $y = 1 + 1 = 2$

When $x = 3$, $y = 1 + 3 = 4$

The solutions are $x = 1$, $y = 2$ and $x = 3$, $y = 4$

Check: $x = 1, y = 2 \Rightarrow 3x^2 - 2y^2 = 3 - 5 = -5$

$x = 3, y = 4 \Rightarrow 3x^2 - 2y^2 = 27 - 32 = -5$

(iii) $p^2 + pq = 2$ (1)

$q - p = 3$ (2)

(2) $\Rightarrow q = 3 + p$

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Substituting into (1): $p^2 + p(3+p) = 2$

$p^2 + 3p + p^2 = 2$

$2p^2 + 3p - 2 = 0$

$(2p-1)(p+2) = 0$

$p = \frac{1}{2}$ or $p = -2$

When $p = \frac{1}{2}, q = 3 + \frac{1}{2} = \frac{7}{2}$

When $p = -2, q = 3 - 2 = 1$

The solutions are $p = \frac{1}{2}, q = \frac{7}{2}$ and $p = -2, q = 1$.

Check: $p = \frac{1}{2}, q = \frac{7}{2} \Rightarrow p^2 + pq = \frac{1}{4} + \frac{7}{4} = 2$

$p = -2, q = 1 \Rightarrow p^2 + pq = 4 - 2 = 2$

(iv) $8a^2 - b^2 = 2$ (1)

$2a + b = 1$ (2)

(2) $\Rightarrow b = 1 - 2a$

Substituting into (1): $8a^2 - (1-2a)^2 = 2$

$8a^2 - (1 - 4a + 4a^2) = 2$

$8a^2 - 1 + 4a - 4a^2 = 2$

$4a^2 + 4a - 3 = 0$

$(2a+3)(2a-1) = 0$

$a = -\frac{3}{2}$ or $a = \frac{1}{2}$

When $a = -\frac{3}{2}, b = 1 - 2 \times -\frac{3}{2} = 1 + 3 = 4$

When $a = \frac{1}{2}, b = 1 - 2 \times \frac{1}{2} = 1 - 1 = 0$

The solutions are $a = -\frac{3}{2}, b = 4$ and $a = \frac{1}{2}, b = 0$.

Check: $a = -\frac{3}{2}, b = 4 \Rightarrow 8a^2 - b^2 = 8 \times \frac{9}{4} - 16 = 18 - 16 = 2$ \

$a = \frac{1}{2}, b = 0 \Rightarrow 8a^2 - b^2 = 8 \times \frac{1}{4} - 0 = 2$

2. (i) $y = 9 - x \Rightarrow x^2 - 3x(9-x) + 2(9-x)^2 = 0$

$\Rightarrow 2x^2 - 21x + 54 = 0$

$\Rightarrow (2x-9)(x-6) = 0$

$\Rightarrow x = \frac{9}{2}, y = \frac{9}{2}$ or $x = 6, y = 3$

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(ii) $y = \frac{8}{x} \Rightarrow 3x - \frac{8}{x} = 10$

$\Rightarrow 3x^2 - 8 = 10x$

$\Rightarrow 3x^2 - 10x - 8 = 0$

$\Rightarrow (x-4)(3x+2) = 0$

$\Rightarrow x = 4, y = 2 \text{ or } x = -\frac{2}{3}, y = -12$

(iii) $y = 4x$

$3y^2 - 2xy = 160$

$\Rightarrow 3(4x)^2 - 2x(4x) = 160$

$\Rightarrow 40x^2 = 160$

$\Rightarrow x = 2, y = 8 \text{ or } x = -2, y = -8$

3. (i) $y = 3x - 2$

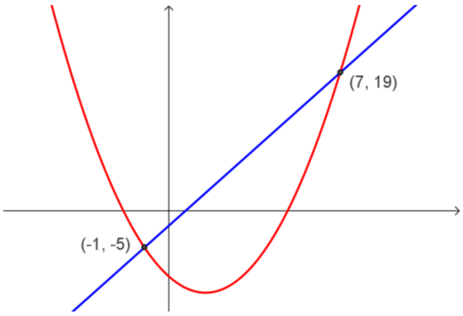
$y = x^2 - 3x - 9$

$\Rightarrow 3x - 2 = x^2 - 3x - 9$

$\Rightarrow x^2 - 6x - 7 = 0$

$\Rightarrow (x-7)(x+1) = 0$

$\Rightarrow x = 7, y = 19 \text{ or } x = -1, y = -5$



(ii) $y + 2x = 3$

$y = 6 + 4x - x^2$

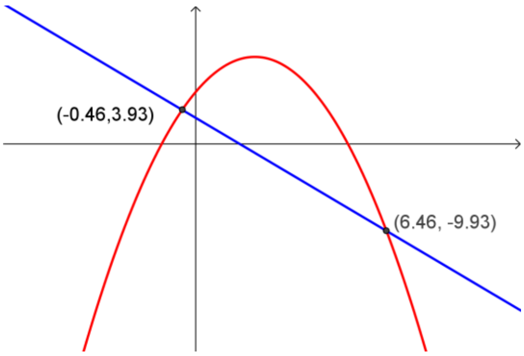
$\Rightarrow -2x + 3 = 6 + 4x - x^2$

$\Rightarrow x^2 - 6x - 3 = 0$

$\Rightarrow x = \frac{6 \pm \sqrt{36+12}}{2}$

$\Rightarrow x = 6.46, y = -9.93$

$\text{or } x = -0.46, y = 3.93$



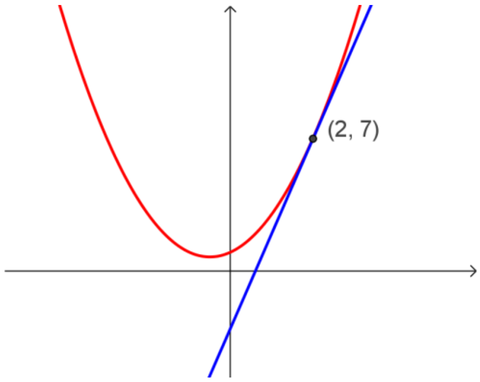
4. (i) $x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$

so vertex is $\left(-\frac{1}{2}, \frac{3}{4}\right)$

(ii) $\left.\begin{array}{l} y = x^2 + x + 1 \\ y = 5x - 3 \end{array}\right\}$ cross when

$$x^2 + x + 1 = 5x - 3$$
$$\Rightarrow x^2 - 4x + 4 = 0$$
$$\Rightarrow \quad (x-2)^2 = 0$$
$$\Rightarrow \quad x = 2 \quad \text{(twice!)}$$

(iii) When $x = 2$, then $y = 7$



From the graph, there is exactly one 'crossing' point, at (2 , 7) so the straight line is a tangent to the quadratic graph.

5. $x^2 + kx + 6 = 2x - 3$

$x^2 + (k-2)x + 9 = 0$

$a = 1, b = k - 2, c = 9$

If the graphs touch, discriminant = 0

$(k-2)^2 - 4 \times 1 \times 9 = 0$

$(k-2)^2 = 36$

$k - 2 = \pm 6$

$k = 2 \pm 6$

$k = -4$ or 8