Section 1: Quadratic graphs and equations

Solutions to Exercise level 1

1. (i) $x^2+5x+6 = x^2+3x+2x+6$
$= x(x+3)+2(x+3)$
$= (x+2)(x+3)$

(ii) $x^2+x-12 = x^2+4x-3x-12$
$= x(x+4)-3(x+4)$
$= (x-3)(x+4)$

(iii) $x^2-9 = (x+3)(x-3)$

(iv) $x^2-6x+8 = x^2-2x-4x+8$
$= x(x-2)-4(x-2)$
$= (x-4)(x-2)$

(v) $2x^2+3x+1 = 2x^2+x+2x+1$
$= x(2x+1)+1(2x+1)$
$= (x+1)(2x+1)$

(vi) $3x^2+x-2 = 3x^2+3x-2x-2$
$= 3x(x+1)-2(x+1)$
$= (3x-2)(x+1)$

(vii) $4x^2-8x+3 = 4x^2-2x-6x+3$
$= 2x(2x-1)-3(2x-1)$
$= (2x-3)(2x-1)$

(viii) $4x^2-25 = (2x+5)(2x-5)$

(ix) $6x^2-x-12 = 6x^2+8x-9x-12$
$= 2x(3x+4)-3(3x+4)$
$= (2x-3)(3x+4)$

2. (i) $x^2-4x = x(x-4)$

(ii) $x^2-17x-60 = (x-20)(x+3)$

(iii) $x^2 + 4(x+1) = x^2 + 4x + 4$
$= (x+2)^2$

(iv) $3x^2 - 11x + 6 = (3x-2)(x-3)$

3. (i) $x^2 + 4x + 3 = 0$
$(x+3)(x+1) = 0$
$x = -3$ or $x = -1$

(ii) $x^2 + 5x - 6 = 0$
$(x+6)(x-1) = 0$
$x = -6$ or $x = 1$

(iii) $x^2 - 6x + 8 = 0$
$(x-2)(x-4) = 0$
$x = 2$ or $x = 4$

(iv) $x^2 - 7x - 18 = 0$
$(x-9)(x+2) = 0$
$x = 9$ or $x = -2$

(v) $2x^2 + 5x + 3 = 0$
$(2x+3)(x+1) = 0$
$x = -\frac{3}{2}$ or $x = -1$

(vi) $2x^2 + x - 6 = 0$
$(2x-3)(x+2) = 0$
$x = \frac{3}{2}$ or $x = -2$

4. (i) Line of symmetry is $x = 4$
Vertex (minimum point) is (4, 1)

(ii) Line of symmetry is $x = -2$
Vertex (minimum point) is (-2, -3)

(iii) Line of symmetry is $x = \frac{1}{2}$
Vertex (minimum point) is $\left(\frac{1}{2}, -5\right)$

(iv) Line of symmetry is $x = -1$
Vertex (maximum point) is (-1, 3).

5. Minimum point is (-1, 2)
Equation of graph is $y = (x+1)^2 + 2$
$= x^2 + 2x + 1 + 2$
$= x^2 + 2x + 3$

6. Maximum point is (2, 5)
Equation of graph is $y = 5 - (x-2)^2$
$= 5 - (x^2 - 4x + 4)$
$= 5 - x^2 + 4x - 4$
$= -x^2 + 4x + 1$

7. (i) $x^2 + 2x - 3 = (x+1)^2 - 1^2 - 3$
$= (x+1)^2 - 4$

(ii) $x^2 - 6x + 1 = (x-3)^2 - 3^2 + 1$
$= (x-3)^2 - 8$

(iii) $x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1$
$= \left(x + \frac{1}{2}\right)^2 - \frac{1}{4} + 1$
$= \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$

(iv) $-x^2 + 5x = -\left(x^2 - 5x\right)$
$= -\left(\left(x - \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2\right)$
$= -\left(x - \frac{5}{2}\right)^2 + \frac{25}{4}$

(v) $2x^2 + 4x + 3 = 2\left(x^2 + 2x\right) + 3$
$= 2\left((x+1)^2 - 1^2\right) + 3$
$= 2(x+1)^2 - 2 + 3$
$= 2(x+1)^2 + 1$

(vi) $3x^2+8x-2=3\left(x^2+\frac{8}{3}x\right)-2$

$=3\left(\left(x+\frac{4}{3}\right)^2-\left(\frac{4}{3}\right)^2\right)-2$

$=3\left(x+\frac{4}{3}\right)^2-3\times\frac{16}{9}-2$

$=3\left(x+\frac{4}{3}\right)^2-\frac{16}{3}-2$

$=3\left(x+\frac{4}{3}\right)^2-\frac{22}{3}$

8. (i) $(-1,-4)$ minimum

(ii) $(3,-8)$ minimum

(iii) $\left(-\frac{1}{2},\frac{3}{4}\right)$ minimum

(iv) $\left(\frac{5}{2},\frac{25}{4}\right)$ maximum

(v) $(-1,\ 1)$ minimum

(vi) $\left(-\frac{4}{3},-\frac{22}{3}\right)$ minimum