Section 2: The quadratic formula

Solutions to Exercise level 3

1. The stone passes the clifftop when $h = 0$,

$$h = 0 \Rightarrow 0 = 20t - 5t^2$$
$$\Rightarrow 5t(t - 4) = 0$$
$$\Rightarrow t = 0, t = 4$$

So the stone passes the clifftop on the way downwards after 4 seconds.

The stone reaches the sea when $h = -50$,

$$h = -50 \Rightarrow 5t^2 - 20t - 50 = 0$$
$$\Rightarrow \quad t^2 - 4t - 10 = 0$$
$$\Rightarrow \quad t = \frac{4 \pm \sqrt{16 + 40}}{2}$$
$$\Rightarrow \quad t = 5.74, -1.74$$

The stone hits the sea after 5.74 seconds (to 3 sig. figs.).

The negative root can be interpreted as the time before the stone was thrown when it should have been thrown from sea level to follow the same path.
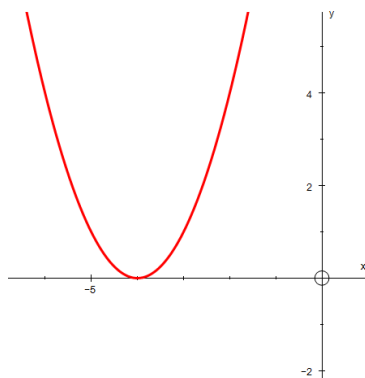
2. (i) $x^2 + 8x + c = (x + 4)^2 + (c - 16)$

   (ii) If the equation has real roots, then the vertex of the graph must be on or below the x – axis, so $\quad c - 16 \le 0$
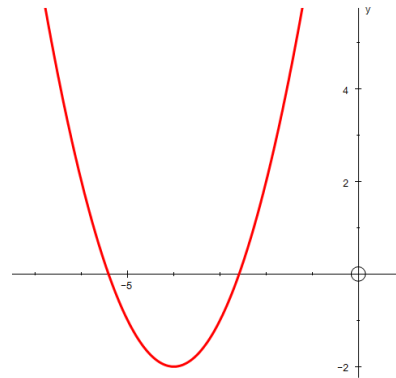 $$\Rightarrow \quad c \le 16$$

   (iii) If the equation has unequal real roots, then the vertex must be strictly below the x – axis, so $\quad c - 16 < 0$
   $$\Rightarrow \quad c < 16$$

   (iv) Two real equal roots $\qquad\qquad$ Two real unequal roots
   $\qquad\quad c = 16 \qquad\qquad\qquad\qquad\quad$ e.g. $c = 14$

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(v) In the two graphs, the vertex in each case lies on the line $x = -4$ and this will be the case for all values of c. So there will never be a graph with two intercepts with the x-axis which are both positive.

3. (i) In the diagram

$$2x + 2y = 184$$

$$\Rightarrow \quad y = 92 - x$$

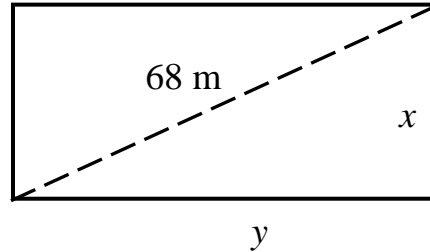By Pythagoras' Theorem,

$$x^2 + y^2 = 68^2$$

$$\Rightarrow \quad x^2 + (92 - x)^2 = 68^2$$
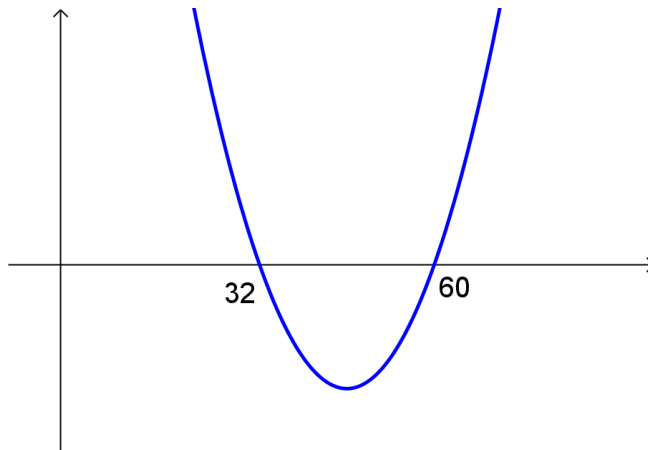
$$\Rightarrow 2x^2 - 184x + 3840 = 0$$

$$\Rightarrow \quad x^2 - 92x + 1920 = 0$$

$$\Rightarrow \quad (x - 32)(x - 60) = 0$$

So the car park measures 32 metres by 60 metres.

The rectangle with diagonal labelled "68 m", right side labelled x, and bottom labelled y.

(ii)

The intersections give both dimensions of the car park.