Section 2: The quadratic formula

Solutions to Exercise level 2

1. (i) $x^2 + 2x - 2 = 0$

$a = 1, b = 2, c = -2$

Discriminant $= b^2 - 4ac = 2^2 - 4 \times 1 \times -2 = 4 + 8 = 12$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{12}}{2} = \frac{-2 \pm 2\sqrt{3}}{2} = -1 \pm \sqrt{3}$$

(ii) $x^2 - 3x + 5 = 0$

$a = 1, b = -3, c = 5$

Discriminant $= b^2 - 4ac = (-3)^2 - 4 \times 1 \times 5 = 9 - 20 = -11$

The discriminant is negative so there are no real roots.

(iii) $2x^2 + x - 4 = 0$

$a = 2, b = 1, c = -4$

Discriminant $= b^2 - 4ac = 1^2 - 4 \times 2 \times -4 = 1 + 32 = 33$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{33}}{2 \times 2} = \frac{-1 \pm \sqrt{33}}{4}$$

(iv) $2x^2 - 5x - 12 = 0$

$a = 2, b = -5, c = -12$

Discriminant $= b^2 - 4ac = (-5)^2 - 4 \times 2 \times -12 = 25 + 96 = 121$

Since the discriminant is a perfect square, then it is possible to factorise.

$(2x + 3)(x - 4) = 0$

$x = -\frac{3}{2}$ or $x = 4$

(v) $x^2 - 5x - 3 = 0$

$a = 1, b = -5, c = -3$

Discriminant $= b^2 - 4ac = (-5)^2 - 4 \times 1 \times -3 = 25 + 12 = 37$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{37}}{2}$$

(vi) $3x^2 + x + 1 = 0$

$a = 3, b = 1, c = 1$

Discriminant $= b^2 - 4ac = 1^2 - 4 \times 3 \times 1 = 1 - 12 = -11$

The discriminant is negative so there are no real roots.

(vii) $4x^2 + 12x + 9 = 0$

$a = 4, b = 12, c = 9$

Discriminant $= b^2 - 4ac = 12^2 - 4 \times 4 \times 9 = 144 - 144 = 0$
Since the discriminant is zero, there is a repeated root and the equation can be factorised.

$(2x+3)^2 = 0$

$x = -\frac{3}{2}$

(viii) $4x^2 + 10x + 5 = 0$

$a = 4, b = 10, c = 5$

Discriminant $= b^2 - 4ac = 10^2 - 4 \times 4 \times 5 = 100 - 80 = 20$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-10 \pm \sqrt{20}}{2 \times 4} = \frac{-10 \pm 2\sqrt{5}}{8} = \frac{-5 \pm \sqrt{5}}{4}$$

2. (i) $x = \dfrac{3}{x} - 1$

Multiplying through by x: $x^2 = 3 - x$

$$x^2 + x - 3 = 0$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times -3}}{2 \times 1}$$

$$= \frac{-1 \pm \sqrt{13}}{2}$$

(ii) $6\sqrt{x} - 7 = x$

Substituting $\sqrt{x} = y$ gives $6y - 7 = y^2$

$$y^2 - 6y + 7 = 0$$

$$y = \frac{6 \pm \sqrt{6^2 - 4 \times 1 \times 7}}{2 \times 1}$$

$$= \frac{6 \pm \sqrt{8}}{2}$$

$$= \frac{6 \pm 2\sqrt{2}}{2}$$

$$= 3 \pm \sqrt{2}$$

$$x = y^2 = \left(3 \pm \sqrt{2}\right)^2$$

$$= 9 \pm 6\sqrt{2} + 2$$

$$= 11 \pm 6\sqrt{2}$$

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3. Surface area of cylinder $= 2\pi r^2 + 2\pi rh$
$$= 2\pi r^2 + 40\pi r$$
$$2\pi r^2 + 40\pi r = 300$$
$$\pi r^2 + 20\pi r - 150 = 0$$
$$r = \frac{-20\pi \pm \sqrt{(20\pi)^2 - 4 \times \pi \times -150}}{2 \times \pi}$$
$$= 2.16 \text{ or } -22.2$$
Since the radius must be positive, $r = 2.16$.

4. If there is a repeated root, the discriminant is zero.
$a = 1,\ b = (3k+1),\ c = (4k+13)$
$$b^2 - 4ac = 0$$
$$(3k+1)^2 - 4(4k+13) = 0$$
$$9k^2 + 6k + 1 - 16k - 52 = 0$$
$$9k^2 - 10k - 51 = 0$$
$$(k-3)(9k+17) = 0$$
$$k = 3 \text{ or } k = -\tfrac{17}{9}$$

5. If there is at least one real root, the discriminant is greater than or equal to zero.
$a = 2,\ b = -5,\ c = k$
$$b^2 - 4ac \geq 0$$
$$(-5)^2 - 4 \times 2 \times k \geq 0$$
$$25 - 8k \geq 0$$
$$k \leq \tfrac{25}{8}$$
The greatest possible value of k is $\tfrac{25}{8}$.