Section 2: Indices

Solutions to Exercise level 3 (Extension)

1. (i) $T^2 = kr^3$

(ii) $T = \sqrt{k}r^{\frac{3}{2}}$ and $r = \dfrac{T^{\frac{2}{3}}}{k^{\frac{1}{3}}}$

(iii) $T^2 = kr^3 \Rightarrow k = \dfrac{T^2}{r^3}$

$$\Rightarrow k = \frac{(27.3)^2}{382300^3} = 1.334 \times 10^{-14}$$

(iv) $T = 1 \Rightarrow$ geostationary satellite radius $= 42200$ (3 s.f.)

(v) $r = 110000 \Rightarrow$ circumpolar satellite period $= 4.21$ days

(vi) Period $= 1.5$ hours $\Rightarrow T = 0.0625$ days

$$\Rightarrow R = \frac{(0.0625)^{\frac{2}{3}}}{(1.334 \times 10^{-14})^{\frac{1}{3}}} = 6640 \text{ km}$$

so the space station is $6640 - 6371 \approx 270$ km above the ground.

2. (i) In the diagram, $(R+h)^2 = R^2 + d^2$

$$\Rightarrow R^2 + 2Rh + h^2 = R^2 + d^2$$
$$\Rightarrow d^2 = h(2R + h)$$
$$\Rightarrow d = \sqrt{h}\sqrt{2R + h}$$

(ii) If h is very small in comparison with R, then the term in h^2 can be neglected in comparison with the term in Rh, leading to $d = \sqrt{2Rh}$

h metres $= \dfrac{h}{1000}$ kilometres, so $d = \sqrt{2 \times 6371 \times \dfrac{h}{1000}}$

$$\approx 3.57\sqrt{h}$$

(iii) (a) $h = 2 \Rightarrow d = 5.048$ km

(b) $h = 100 \Rightarrow d = 35.71$ km

(c) $h = 1000 \Rightarrow d = 112.9$ km

(d) $h = 10000 \Rightarrow d = 357.1$ km

(iv) Using the formula:

	Penvose		Tredeen	
	High tide	Low tide	High tide	Low tide
height (m)	59	65	62	68
distance(km)	27.42	28.78	28.11	29.43

so 27.42 + 28.11) < distance apart < (28.78 + 29.43)
55.3 km < distance apart < 58.22 km

3. (i) $-2 = k(30+10)^{\frac{5}{3}}$

$\Rightarrow k = -\dfrac{2}{40^{\frac{5}{3}}} = -0.004275$

(ii) $t = 0,\ \theta = 40 \Rightarrow 0 = -\left(\dfrac{3}{2k}\right)(40)^{-\frac{2}{3}} + C$

$\Rightarrow C = \left(\dfrac{3}{2k}\right)(40)^{-\frac{2}{3}}$

$= -\left(\dfrac{3(40)^{\frac{5}{3}}}{4}\right)(40)^{-\frac{2}{3}}$

$= -\frac{3}{4}(40)$

So $t = \frac{3}{4}(40)^{\frac{5}{3}}(\theta+10)^{-\frac{2}{3}} - \frac{3}{4}(40)$

and when $\theta = 0$, $t = \frac{3}{4}(40)^{\frac{5}{3}}(10)^{-\frac{2}{3}} - \frac{3}{4}(40)$

$= \frac{3}{4}(40)\left(\dfrac{40^{\frac{2}{3}}}{10^{\frac{2}{3}}} - 1\right)$

$= 30(4^{\frac{2}{3}} - 1)$

$= 45.6$ minutes