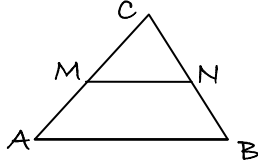


Section 1: Vectors in three dimensions

Solutions to Exercise level 3

1.



$$\begin{aligned}
 \overrightarrow{MN} &= \overrightarrow{MA} + \overrightarrow{AB} + \overrightarrow{BN} \\
 &= \frac{1}{2}\overrightarrow{CA} + \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} \\
 &= \frac{1}{2}(\underline{a} - \underline{c}) + (\underline{b} - \underline{a}) + \frac{1}{2}(\underline{c} - \underline{b}) \\
 &= \frac{1}{2}\underline{a} - \frac{1}{2}\underline{c} + \underline{b} - \underline{a} + \frac{1}{2}\underline{c} - \frac{1}{2}\underline{b} \\
 &= -\frac{1}{2}\underline{a} + \frac{1}{2}\underline{b} \\
 &= \frac{1}{2}(\underline{b} - \underline{a})
 \end{aligned}$$

$$2. \quad \overrightarrow{AB} = \underline{b} - \underline{a} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 0 \end{pmatrix}$$

$$\overrightarrow{AD} = \underline{d} - \underline{a} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix}$$

$$\overrightarrow{AE} = \underline{e} - \underline{a} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$$

So \overrightarrow{AB} , \overrightarrow{AD} and \overrightarrow{AE} are all at right angles to each other and all have magnitude 2, so they are the edges of a cube.

Edexcel A level Maths Vectors 1 Exercise solutions

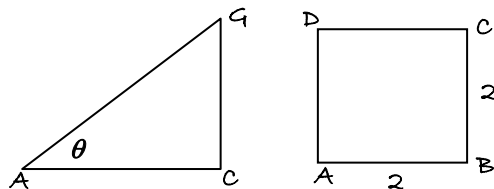
$$\overrightarrow{AG} = \overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{AE} = \begin{pmatrix} -2 \\ -2 \\ -2 \end{pmatrix}$$

$$\underline{g} - \underline{a} = \begin{pmatrix} -2 \\ -2 \\ -2 \end{pmatrix}$$

$$\underline{g} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ -2 \end{pmatrix}$$

$$\underline{g} = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$$

$$G = (-1, -1, -1)$$



$$AC^2 = 2^2 + 2^2$$

$$AC = 2\sqrt{2}$$

$$\overrightarrow{AG} = \begin{pmatrix} -2 \\ -2 \\ -2 \end{pmatrix} \text{ so } AG^2 = \sqrt{2^2 + 2^2 + 2^2} = \sqrt{12} = 2\sqrt{3}$$

$$\cos \theta = \frac{AC}{AG} = \frac{2\sqrt{2}}{2\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\theta = 35.3^\circ$$