Section 1: Vectors in three dimensions

Solutions to Exercise level 2

1. $\overrightarrow{XY} = \begin{pmatrix} -1 \\ 8 \\ -8 \end{pmatrix} - \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 12 \\ -8 \end{pmatrix} = 4\begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$

$\overrightarrow{XZ} = \begin{pmatrix} 6 \\ -13 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ -9 \\ 6 \end{pmatrix} = -3\begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$

$\overrightarrow{XZ}$ is a scalar multiple of $\overrightarrow{XY}$, so X, Y and Z are collinear.

2. $\overrightarrow{AB} = \begin{pmatrix} 8 \\ 7 \\ -9 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ -12 \end{pmatrix}$ $\quad \overrightarrow{BC} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} - \begin{pmatrix} 8 \\ 7 \\ -9 \end{pmatrix} = \begin{pmatrix} -6 \\ -10 \\ 8 \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \\ -4 \end{pmatrix}$

$|\overrightarrow{AB}| = \sqrt{9^2 + 5^2 + (-12)^2} = \sqrt{250}$

$|\overrightarrow{BC}| = \sqrt{(-6)^2 + (-10)^2 + 8^2} = \sqrt{200}$

$|\overrightarrow{AC}| = \sqrt{3^2 + (-5)^2 + (-4)^2} = \sqrt{50}$

so $|\overrightarrow{BC}|^2 + |\overrightarrow{AC}|^2 = 200 + 50 = 250 = |\overrightarrow{AB}|^2$

so by Pythagoras' theorem, ABC has a right angle at C

so vectors BC and AC are perpendicular.

$$\begin{aligned}
\text{Area of triangle} &= \tfrac{1}{2}|BC||AC| \\
&= \tfrac{1}{2}\sqrt{200}\sqrt{50} \\
&= \tfrac{1}{2} \times 10\sqrt{2} \times 5\sqrt{2} \\
&= 25 \times 2 \\
&= 50 \text{ square units}
\end{aligned}$$

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3. $p\begin{pmatrix}3\\1\\2\end{pmatrix}+q\begin{pmatrix}2\\-1\\3\end{pmatrix}=\begin{pmatrix}4\\1\\k\end{pmatrix}$

$3p+2q=4 \quad (1)$

$p-q=1 \quad (2)$

$2p+3q=k \quad (3)$

$(2) \Rightarrow p=1+q$

Substituting into (1) gives $3(1+q)+2q=4$

$$3+3q+2q=4$$
$$5q=1$$
$$q=\tfrac{1}{5}$$
$$p=\tfrac{6}{5}$$

$(3) \Rightarrow k=2\times\tfrac{6}{5}+3\times\tfrac{1}{5}=3$

4. $\overrightarrow{FG}=\begin{pmatrix}3\\-3\\-3\end{pmatrix}$

$\overrightarrow{OH}=\overrightarrow{OE}+\overrightarrow{EH}$

$=\overrightarrow{OE}+\overrightarrow{FG}$

$=\begin{pmatrix}1\\2\\-5\end{pmatrix}+\begin{pmatrix}3\\-3\\-3\end{pmatrix}=\begin{pmatrix}4\\-1\\-8\end{pmatrix}$

so H is (4, -1, -8)

5. $\underset{\sim}{p}=c\begin{pmatrix}2\\-2\\1\end{pmatrix}$

$\left|\underset{\sim}{p}\right|=c\sqrt{2^2+(-2)^2+1^2}=3c$

$24=3c$

$c=8$

$\underset{\sim}{p}=8\begin{pmatrix}2\\-2\\1\end{pmatrix}=\begin{pmatrix}16\\-16\\8\end{pmatrix}$