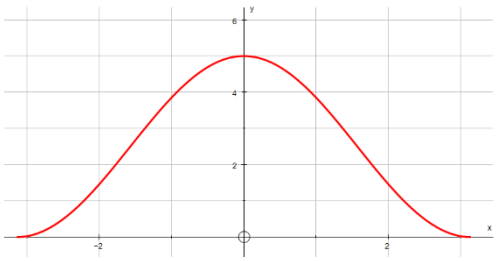
Section 2: Numerical integration

Solutions to Exercise level 3

1. (i) Need minimum value of y (when $x = -1$) to be 0 and maximum value (when $x = 1$) to be 5, so take $a = b = \frac{5}{2}$.

$$y = \frac{5}{2} + \frac{5}{2}\cos x$$

(ii)



(iii) Between $x = 0$ and $x = \pi$:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
y	5	4.6651	3.75	2.5	1.25	0.3349	0

$$\text{Area} \approx \tfrac{1}{2} \times \tfrac{\pi}{6}\left(5 + 0 + 2(4.6651 + 3.75 + 2.5 + 1.25 + 0.3349)\right)$$
$$\approx 7.854$$

(iv) Mass of water $\approx 7.854 \times 2 \times 175$
≈ 2750 tonnes (3 s.f.)

2. (i) $I = \int_0^{\frac{1}{2}} \sqrt[4]{x}\, dx = \left[\frac{4}{5}x^{\frac{5}{4}}\right]_0^{\frac{1}{2}}$
$= 0.3364$ (4 d.p.)

(ii)

x	0	0.125	0.25	0.375	0.5
y	0	0.5946	0.7071	0.7825	0.8409

$$\text{Area} \approx \tfrac{1}{2}(0.125)\left(0 + 0.8409 + 2\left(0.5946 + 0.7071 + 0.7825\right)\right)$$
$$\approx 0.3130$$

(iii) $\text{Percentage error} = \dfrac{0.3364 - 0.3130}{0.3364} \times 100 \approx 6.9\%$

(iv) The graph is very steep near $x = 0$, so the first trapezium is not a good approximation.