

## Section 2: Numerical integration

## Solutions to Exercise level 1

1. (i)  $f(x) = \frac{1}{1+x^2}$

$$h = 0.5$$

$$f_0 = f(0) = 1$$

$$f_1 = f(0.5) = 0.8$$

$$f_2 = f(1) = 0.5$$

$$f_3 = f(1.5) = \frac{1}{3.25}$$

$$f_4 = f(2) = 0.2$$

Using the trapezium rule:

$$\begin{aligned} \text{Area} &\approx \frac{1}{2}h[f_0 + f_4 + 2(f_1 + f_2 + f_3)] \\ &= \frac{1}{2} \times 0.5 \left[ 1 + 0.2 + 2\left(0.8 + 0.5 + \frac{1}{3.25}\right) \right] \\ &= 1.104 \text{ (3 d.p.)} \end{aligned}$$

(ii)  $f(x) = \sqrt{1+x^2}$

$$h = 1$$

$$f_0 = f(0) = 1$$

$$f_1 = f(1) = \sqrt{2}$$

$$f_2 = f(2) = \sqrt{5}$$

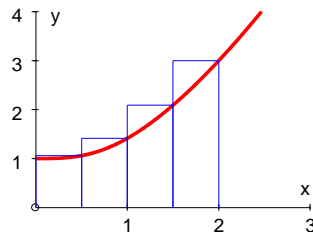
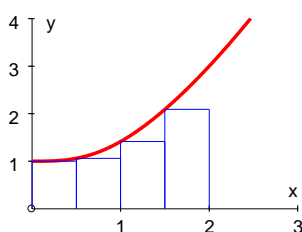
$$f_3 = f(3) = \sqrt{10}$$

Using the trapezium rule:

$$\begin{aligned} \text{Area} &\approx \frac{1}{2}h[f_0 + f_3 + 2(f_1 + f_2)] \\ &= \frac{1}{2} \times 1 \left[ 1 + \sqrt{10} + 2(\sqrt{2} + \sqrt{5}) \right] \\ &= 5.731 \text{ (3 d.p.)} \end{aligned}$$

2.  $\int_0^2 \sqrt{1+x^3} dx$

$$f(x) = \sqrt{1+x^3}$$



## Edexcel A level Maths Num methods 2 Exercise solns

$$\begin{aligned} \text{(i) Underestimate} &= 0.5(f(0) + f(0.5) + f(1) + f(1.5)) \\ &= 0.5(\sqrt{1} + \sqrt{1.125} + \sqrt{2} + \sqrt{4.375}) \\ &= 2.783 \end{aligned}$$

$$\begin{aligned} \text{Overestimate} &= 0.5(f(0.5) + f(1) + f(1.5) + f(2)) \\ &= 0.5(\sqrt{1.125} + \sqrt{2} + \sqrt{4.375} + \sqrt{9}) \\ &= 3.783 \end{aligned}$$

$$\begin{aligned} \text{(ii) Underestimate} &= 0.25(f(0) + f(0.25) + f(0.5) + \dots + f(1.75)) \\ &= 3.002 \end{aligned}$$

$$\begin{aligned} \text{Overestimate} &= 0.25(f(0.25) + f(0.5) + f(0.75) + \dots + f(2)) \\ &= 3.502 \end{aligned}$$

3. Using 5 ordinates (4 strips):  $h = \frac{3-1}{4} = \frac{1}{2}$

$$\begin{aligned} \int_1^3 f(x) dx &\approx \frac{1}{2} h (y_0 + y_4 + 2(y_1 + y_2 + y_3)) \\ &= \frac{1}{2} \times \frac{1}{2} (8.01 + 3.27 + 2(6.02 + 4.69 + 3.80)) \\ &= 10.075 \end{aligned}$$

4. Using 4 strips of width 0.25 between  $x = 2$  and  $x = 3$

$x$	2	2.25	2.5	2.75	3
$y$	1.9566	2.0917	2.2254	2.3580	2.4892

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times \frac{1}{4} (1.9566 + 2(2.0917 + 2.2254 + 2.3580) + 2.4892) \\ &= 2.2245 \end{aligned}$$