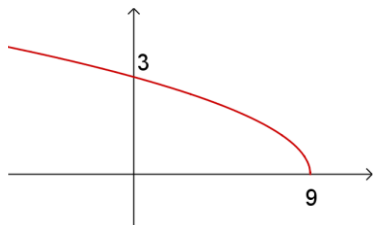


Section 1: Finding areas

Solutions to Exercise level 2

1. (i)



$$(ii) \quad y = \sqrt{9-x} \Rightarrow y^2 = 9-x \Rightarrow x = 9-y^2$$

$$\begin{aligned} \text{Area} &= \int_0^3 x \, dy \\ &= \int_0^3 (9-y^2) \, dy \\ &= \left[9y - \frac{1}{3}y^3 \right]_0^3 \\ &= 27 - 9 - 0 \\ &= 18 \end{aligned}$$

$$2. \quad (i) \quad y = x^3 + 2x^2 - 4x + 1$$

$$\frac{dy}{dx} = 3x^2 + 4x - 4$$

At turning points, $3x^2 + 4x - 4 = 0$

$$(x+2)(3x-2) = 0$$

$$x = -2 \text{ or } \frac{2}{3}$$

From the diagram, the x-coordinate of the maximum point is negative, so the line touches the curve when $x = -2$

$$\text{When } x = -2, \quad y = -8 + 8 + 8 + 1 = 9$$

so the point where the line touches the curve is $(-2, 9)$

and $k = 9$

$$(ii) \quad \text{Where line crosses curve, } x^3 + 2x^2 - 4x + 1 = 9$$

$$x^3 + 2x^2 - 4x - 8 = 0$$

$$(x+2)(x^2-4) = 0$$

$$(x+2)(x+2)(x-2) = 0$$

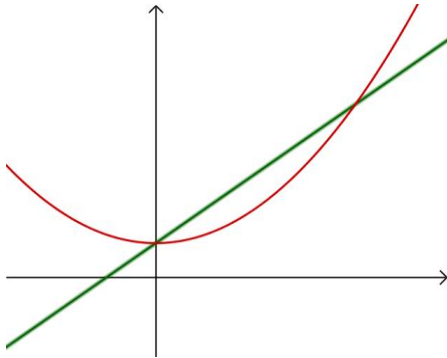
so the line crosses the curve at $x = 2$

and so the coordinates are $(2, 9)$

Edexcel A level Maths Integration 1 Exercise solutions

$$\begin{aligned} \text{(iii) Area} &= \int_{-2}^2 (9 - (x^3 + 2x^2 - 4x + 1)) dx \\ &= \int_{-2}^2 (8 - x^3 - 2x^2 + 4x) dx \\ &= \left[8x - \frac{1}{4}x^4 - \frac{2}{3}x^3 + 2x^2 \right]_{-2}^2 \\ &= 16 - 4 - \frac{16}{3} + 8 - (-16 - 4 + \frac{16}{3} + 8) \\ &= \frac{64}{3} \end{aligned}$$

3.



Intersections of curves: $x^2 + 1 = 2x + 1$

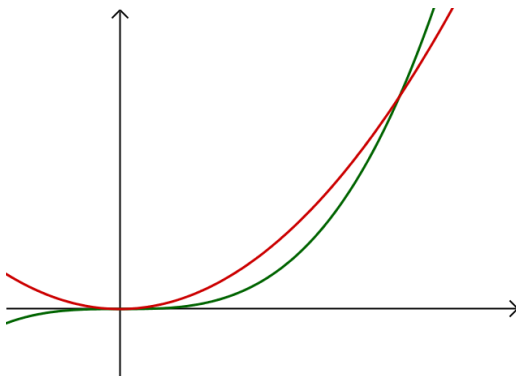
$$\Rightarrow x^2 - 2x = 0$$

$$\Rightarrow x(x - 2) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 2$$

$$\begin{aligned} \text{Area under curve} &= \int_0^2 (2x + 1 - (x^2 + 1)) dx \\ &= \int_0^2 (2x - x^2) dx \\ &= \left[x^2 - \frac{1}{3}x^3 \right]_0^2 \\ &= 4 - \frac{8}{3} - 0 \\ &= \frac{4}{3} \end{aligned}$$

4.



Intersections of curves: $x^2 = x^3 \Rightarrow x = 0 \text{ or } x = 1$

Edexcel A level Maths Integration 1 Exercise solutions

$$\begin{aligned}
 \text{Area} &= \int_0^1 (x^2 - x^3) dx \\
 &= \left[\frac{1}{3}x^3 - \frac{1}{4}x^4 \right]_0^1 \\
 &= \frac{1}{3} - \frac{1}{4} - 0 \\
 &= \frac{1}{12}
 \end{aligned}$$

5. (i) $f(x) = x^3 - x^2 - 4x + 4$

$f(1) = 1 - 1 - 4 + 4 = 0$ so $(x-1)$ is a factor.

$f(x) = (x-1)(x^2 - 4)$

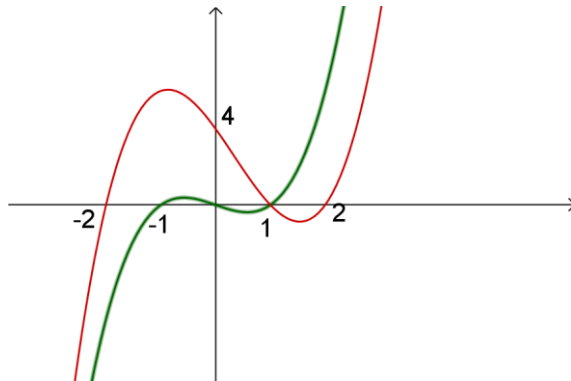
$= (x-1)(x-2)(x+2)$

The graph crosses the x-axis at $(1, 0)$, $(2, 0)$ and $(-2, 0)$, and the y-axis at $(0, 4)$

(ii) $y = x^3 - x$

$= x(x^2 - 1)$

$= x(x+1)(x-1)$



(iii) At intersection points, $x^3 - x^2 - 4x + 4 = x^3 - x$

$x^2 + 3x - 4 = 0$

$(x-1)(x+4) = 0$

$x = 1$ or -4

$$\begin{aligned}
 \text{(iv) Area} &= \int_{-4}^1 (x^3 - x^2 - 4x + 4 - (x^3 - x)) dx \\
 &= \int_{-4}^1 (-x^2 - 3x + 4) dx \\
 &= \left[-\frac{1}{3}x^3 - \frac{3}{2}x^2 + 4x \right]_{-4}^1 \\
 &= -\frac{1}{3} - \frac{3}{2} + 4 - \left(\frac{64}{3} - 24 - 16 \right) \\
 &= \frac{125}{6}
 \end{aligned}$$