Section 3: Further techniques for integration

Solutions to Exercise level 2

1. (i) $\int \frac{1}{\sqrt{2x+1}}\,dx$

   This is a simple function of $2x + 1$, so this integration can be done by inspection.
   Alternatively, the substitution $u = 2x + 1$ may be used.

   (ii) $\int \frac{1}{x^2-4}\,dx$

   This integral can be done using partial fractions.

   (iii) $\int \frac{x}{x^2-4}\,dx$

   The numerator is a multiple of the derivative of the denominator, so this integration can be done by inspection.
   Alternatively, the substitution $u = x^2 - 4$ may be used.
   (The integration could also be done using partial fractions, but this is probably less efficient).

2. $\frac{1}{x^2+3x+2} = \frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

   $1 = A(x+2) + B(x+1)$

   Putting $x = -1 \quad \Rightarrow 1 = A$

   Putting $x = -2 \quad \Rightarrow 1 = -B \quad \Rightarrow B = -1$

   $$\int \frac{1}{x^2+3x+2}\,dx = \int \left(\frac{1}{x+1} - \frac{1}{x+2}\right)$$
   $$= \ln|x+1| - \ln|x+2| + c$$
   $$= \ln\left|\frac{x+1}{x+2}\right| + c$$

3. $\frac{1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$

   $1 = Ax(x+1) + B(x+1) + Cx^2$

   Putting $x = 0 \quad \Rightarrow 1 = B \quad \Rightarrow B = 1$

   Putting $x = -1 \quad \Rightarrow 1 = C$

   Equating coefficients of $x^2 \quad \Rightarrow 0 = A + C \quad \Rightarrow A = -C = -1$

Edexcel A level Maths Integration 3 Exercise solutions

$$\int_1^2 \frac{1}{x^2(x+1)}\,dx = \int_1^2 \left(-\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x+1}\right)dx$$
$$= \left[-\ln x - \frac{1}{x} + \ln(x+1)\right]_1^2$$
$$= -\ln 2 - \tfrac{1}{2} + \ln 3 - (0 - 1 + \ln 2)$$
$$= -\ln 2 - \tfrac{1}{2} + \ln 3 + 1 - \ln 2$$
$$= \tfrac{1}{2} + \ln 3 - 2\ln 2$$
$$= \tfrac{1}{2} + \ln\left(\tfrac{3}{4}\right)$$

4. The derivative of x^2-4 is $2x$, so this integral can be done by inspection.

$$\int \frac{x}{\sqrt{x^2-4}}\,dx = \tfrac{1}{2}\int \frac{2x}{\sqrt{x^2-4}}\,dx$$
$$= \tfrac{1}{2}\int 2x\left(x^2-4\right)^{-\frac{1}{2}}\,dx$$
$$= \tfrac{1}{2}\times 2\left(x^2-4\right)^{\frac{1}{2}} + c$$
$$= \sqrt{x^2-4} + c$$

5. Let $u = 1 + x \Rightarrow \dfrac{du}{dx} = 1 \Rightarrow du = dx$

When $x = 0$, $u = 1$

When $x = 1$, $u = 2$

$$\int_0^1 x\sqrt{1+x}\,dx = \int_1^2 (u-1)\sqrt{u}\,du$$
$$= \int_1^2 \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)du$$
$$= \left[\tfrac{2}{5}u^{\frac{5}{2}} - \tfrac{2}{3}u^{\frac{3}{2}}\right]_1^2$$
$$= \left(\tfrac{2}{5}\times 4\sqrt{2} - \tfrac{2}{3}\times 2\sqrt{2}\right) - \left(\tfrac{2}{5} - \tfrac{2}{3}\right)$$
$$= \tfrac{4}{15}\sqrt{2} + \tfrac{4}{15}$$
$$= \tfrac{4}{15}(1+\sqrt{2})$$

6. The derivative of x^2+3x+2 is $2x+3$, so this integral can be done by inspection.

$$\int_0^1 \frac{2x+3}{x^2+3x+2}\,dx = \left[\ln(x^2+3x+2)\right]_0^1$$
$$= \ln 6 - \ln 2$$
$$= \ln 3$$