Section 3: Further techniques for integration

Solutions to Exercise level 1

1. (i) $\dfrac{2}{(x+1)(x-1)} = \dfrac{A}{x+1} + \dfrac{B}{x-1}$

$2 = A(x-1) + B(x+1)$

Let $x = 1 \quad \Rightarrow 2 = 2B \quad \Rightarrow B = 1$

Let $x = -1 \quad \Rightarrow 2 = -2A \quad \Rightarrow A = -1$

$\dfrac{2}{(x+1)(x-1)} = \dfrac{1}{x-1} - \dfrac{1}{x+1}$

(ii) $\displaystyle\int \frac{2}{(x+1)(x-1)}\,dx = \int\left(\frac{1}{x-1} - \frac{1}{x+1}\right)dx$

$= \ln|x-1| - \ln|x+1| + c$

$= \ln\left|\dfrac{x-1}{x+1}\right| + c$

2. (i) $\dfrac{4}{x^2(x-2)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-2}$

$4 = Ax(x-2) + B(x-2) + Cx^2$

Let $x = 0 \quad \Rightarrow 4 = -2B \quad \Rightarrow B = -2$

Let $x = 2 \quad \Rightarrow 4 = 4C \quad \Rightarrow C = 1$

Equating coefficients of $x^2 \Rightarrow 0 = A + C \quad \Rightarrow A = -1$

$\dfrac{4}{x^2(x-2)} = \dfrac{1}{x-2} - \dfrac{1}{x} - \dfrac{2}{x^2}$

(ii) $\displaystyle\int \frac{4}{x^2(x-2)}\,dx = \int\left(\frac{1}{x-2} - \frac{1}{x} - \frac{2}{x^2}\right)dx$

$= \ln|x-2| - \ln|x| + \dfrac{2}{x} + c$

$= \ln\left|\dfrac{x-2}{x}\right| + \dfrac{2}{x} + c$

3. (i) $\dfrac{5}{(2x+1)(x+3)} = \dfrac{A}{2x+1} + \dfrac{B}{x+3}$

$5 = A(x+3) + B(2x+1)$

Let $x = -3 \quad \Rightarrow 5 = -5B \quad \Rightarrow B = -1$

Let $x = -\frac{1}{2} \quad \Rightarrow 5 = \frac{5}{2}A \quad \Rightarrow A = 2$

$\dfrac{5}{(2x+1)(x+3)} = \dfrac{2}{2x+1} - \dfrac{1}{x+3}$

(ii) $\int_0^1 \frac{5}{(2x+1)(x+3)}dx = \int_0^1 \left(\frac{2}{2x+1} - \frac{1}{x+3}\right)dx$

$= \left[\ln|2x+1| - \ln|x+3|\right]_0^1$

$= \ln 3 - \ln 4 - \ln 1 + \ln 3$

$= 2\ln 3 - 2\ln 2$

$= 2\ln \frac{3}{2}$

4. (i) $\frac{3x+5}{(x+1)(x+2)(x+3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+3}$

$3x+5 = A(x+2)(x+3) + B(x+1)(x+3) + C(x+1)(x+2)$

Let $x=-1 \quad \Rightarrow 2 = 2A \quad \Rightarrow A = 1$

Let $x=-2 \quad \Rightarrow -1 = -B \quad \Rightarrow B = 1$

Let $x=-3 \quad \Rightarrow -4 = 2C \quad \Rightarrow C = -2$

$\frac{3x+5}{(x+1)(x+2)(x+3)} = \frac{1}{x+1} + \frac{1}{x+2} - \frac{2}{x+3}$

(ii) $\int_0^2 \frac{3x+5}{(x+1)(x+2)(x+3)}dx = \int_0^2 \left(\frac{1}{x+1} + \frac{1}{x+2} - \frac{2}{x+3}\right)dx$

$= \left[\ln|x+1| + \ln|x+2| - 2\ln|x+3|\right]_0^2$

$= \ln 3 + \ln 4 - 2\ln 5 - \ln 1 - \ln 2 + 2\ln 3$

$= 3\ln 3 + 2\ln 2 - \ln 2 - 2\ln 5$

$= \ln 27 + \ln 2 - \ln 25$

$= \ln \frac{54}{25}$

5. (i) $\frac{16}{(x+1)^2(x-3)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-3}$

$16 = A(x+1)(x-3) + B(x-3) + C(x+1)^2$

Let $x=-1 \quad \Rightarrow 16 = -4B \quad \Rightarrow B = -4$

Let $x=3 \quad \Rightarrow 16 = 16C \quad \Rightarrow C = 1$

Equating coefficients of $x^2 \Rightarrow 0 = A + C \quad \Rightarrow A = -1$

$\frac{16}{(x+1)^2(x-3)} = \frac{1}{x-3} - \frac{1}{x+1} - \frac{4}{(x+1)^2}$

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(ii) $\int_1^2 \frac{16}{(x+1)^2(x-3)}\,dx = \int_1^2 \left(\frac{1}{x-3} - \frac{1}{x+1} - \frac{4}{(x+1)^2}\right)dx$

$= \left[\ln|x-3| - \ln|x+1| + \frac{4}{x+1}\right]_1^2$

$= \ln 1 - \ln 3 + \frac{4}{3} - \ln 2 + \ln 2 - 2$

$= -\ln 3 - \frac{2}{3}$