Section 4: Integration by parts

Solutions to Exercise level 1

1. (i) Let $u = x \Rightarrow \frac{du}{dx} = 1$

$$\text{Let } \frac{dv}{dx} = e^{2x} \Rightarrow v = \tfrac{1}{2}e^{2x}$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int xe^{2x}dx = x \times \tfrac{1}{2}e^{2x} - \int \tfrac{1}{2}e^{2x}dx$$

$$= \tfrac{1}{2}xe^{2x} - \tfrac{1}{4}e^{2x} + c$$

(ii) Let $u = 1 - 2x \Rightarrow \frac{du}{dx} = -2$

$$\text{Let } \frac{dv}{dx} = \sin x \Rightarrow v = -\cos x$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int (1-2x)\sin x\, dx = (1-2x) \times -\cos x - \int (-\cos x \times -2)dx$$

$$= -(1-2x)\cos x - 2\int \cos x\, dx$$

$$= -(1-2x)\cos x - 2\sin x + c$$

2. (i) Let $u = 2x \Rightarrow \frac{du}{dx} = 2$

$$\text{Let } \frac{dv}{dx} = e^{-3x} \Rightarrow v = -\tfrac{1}{3}e^{-3x}$$

$$\int_a^b u\frac{dv}{dx}dx = [uv]_a^b - \int_a^b v\frac{du}{dx}dx$$

$$\int_0^1 2xe^{-3x}dx = \left[2x \times -\tfrac{1}{3}e^{-3x}\right]_0^1 - \int_0^1 \left(2 \times -\tfrac{1}{3}e^{-3x}\right)dx$$

$$= \left[-\tfrac{2}{3}xe^{-3x}\right]_0^1 + \tfrac{2}{3}\left[-\tfrac{1}{3}e^{-3x}\right]_0^1$$

$$= -\tfrac{2}{3}e^{-3} - \tfrac{2}{9}e^{-3} + \tfrac{2}{9}$$

$$= \tfrac{2}{9} - \tfrac{8}{9}e^{-3}$$

(ii) Let $u = 2x + 1 \Rightarrow \frac{du}{dx} = 2$

$$\text{Let } \frac{dv}{dx} = \cos x \Rightarrow v = \sin x$$

$$\int_a^b u\frac{dv}{dx}dx = [uv]_a^b - \int_a^b v\frac{du}{dx}dx$$

Edexcel A level Maths Integration 4 Exercise solutions

$$\begin{aligned}\int_0^{\frac{\pi}{2}}(2x+1)\cos x\,dx &= \left[(2x+1)\sin x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 2\sin x\,dx \\ &= \left[(2x+1)\sin x + 2\cos x\right]_0^{\frac{\pi}{2}} \\ &= (\pi+1)\sin\tfrac{\pi}{2} + 2\cos\tfrac{\pi}{2} - \sin 0 - 2\cos 0 \\ &= \pi + 1 - 2 \\ &= \pi - 1\end{aligned}$$