

## Section 2: Integration by substitution

## Solutions to Exercise level 1

$$1. \quad u = 2x + 3 \Rightarrow \frac{du}{dx} = 2 \Rightarrow dx = \frac{1}{2} du$$

$$\begin{aligned} \int (2x + 3)^3 dx &= \int u^3 \times \frac{1}{2} du \\ &= \frac{1}{2} \times \frac{1}{4} u^4 + c \\ &= \frac{1}{8} (2x + 3)^4 + c \end{aligned}$$

$$2. \quad \text{Let } u = 3 - 2x \Rightarrow \frac{du}{dx} = -2 \Rightarrow dx = -\frac{1}{2} du$$

$$\text{When } x = -1, u = 5$$

$$\text{When } x = 1, u = 1$$

$$\begin{aligned} \int_{-1}^1 (3 - 2x)^4 dx &= \int_5^1 u^4 \times -\frac{1}{2} du \\ &= \left[ -\frac{1}{2} \times \frac{1}{5} u^5 \right]_5^1 \\ &= -\frac{1}{10} (1^5 - 5^5) \\ &= -\frac{1}{10} \times -3124 \\ &= 312.4 \end{aligned}$$

$$3. \quad \text{Let } u = 1 + 4x \Rightarrow \frac{du}{dx} = 4 \Rightarrow dx = \frac{1}{4} du$$

$$\text{When } x = 0, u = 1$$

$$\text{When } x = 2, u = 9$$

$$\begin{aligned} \int_0^2 \sqrt{1 + 4x} dx &= \int_1^9 u^{\frac{1}{2}} \times \frac{1}{4} du \\ &= \left[ \frac{1}{4} \times \frac{2}{3} u^{\frac{3}{2}} \right]_1^9 \\ &= \frac{1}{6} (9^{\frac{3}{2}} - 1^{\frac{3}{2}}) \\ &= \frac{1}{6} (27 - 1) \\ &= \frac{13}{3} \end{aligned}$$

$$\begin{aligned} 4. \quad (i) \int_1^2 \frac{1}{x} dx &= [\ln|x|]_1^2 \\ &= \ln 2 - \ln 1 \\ &= \ln 2 \end{aligned}$$

## Edexcel A level Maths Integration 2 Exercise solutions

$$\begin{aligned} \text{(ii)} \int_{-3}^{-1} \frac{1}{x} dx &= [\ln|x|]_{-3}^{-1} \\ &= \ln 1 - \ln 3 \\ &= -\ln 3 \end{aligned}$$

$$\begin{aligned} 5. \text{ (i)} \int_0^2 e^{3x} dx &= \left[ \frac{1}{3} e^{3x} \right]_0^2 \\ &= \frac{1}{3} (e^6 - 1) \end{aligned}$$

$$\begin{aligned} \text{(ii)} \int_0^{\pi/4} \cos 2x dx &= \left[ \frac{1}{2} \sin 2x \right]_0^{\pi/4} \\ &= \frac{1}{2} (\sin \frac{\pi}{2} - 0) \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} 6. \int_0^{\pi/4} (3 \cos x - \sin x) dx &= [3 \sin x + \cos x]_0^{\pi/4} \\ &= (3 \sin \frac{\pi}{4} + \cos \frac{\pi}{4}) - (3 \sin 0 + \cos 0) \\ &= \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 \\ &= \frac{4}{\sqrt{2}} - 1 \\ &= 2\sqrt{2} - 1 \end{aligned}$$

$$7. u = x^3 + 2 \Rightarrow \frac{du}{dx} = 3x^2 \Rightarrow dx = \frac{1}{3x^2} du$$

$$\begin{aligned} \int x^2 (x^3 + 2)^3 dx &= \int x^2 u^3 \times \frac{1}{3x^2} du \\ &= \frac{1}{3} \int u^3 du \\ &= \frac{1}{3} \times \frac{1}{4} u^4 + c \\ &= \frac{1}{12} (x^3 + 2)^4 + c \end{aligned}$$

$$8. u = x + x^2 \Rightarrow \frac{du}{dx} = 1 + 2x \Rightarrow dx = \frac{1}{1 + 2x} du$$

$$\begin{aligned} \int \frac{1 + 2x}{\sqrt{x + x^2}} dx &= \int \frac{1 + 2x}{\sqrt{u}} \times \frac{1}{1 + 2x} du \\ &= \int u^{-\frac{1}{2}} du \\ &= 2u^{\frac{1}{2}} + c \\ &= 2\sqrt{x + x^2} + c \end{aligned}$$

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$$\begin{aligned}9. (i) \int 2 \sin^2 x \, dx &= \int 2 \times \frac{1}{2} (1 - \cos 2x) \, dx \\ &= \int (1 - \cos 2x) \, dx \\ &= x - \frac{1}{2} \sin 2x + c\end{aligned}$$

$$\begin{aligned}(ii) \int 3 \cos^3 x \, dx &= \int 3 \cos x (1 - \sin^2 x) \, dx \\ &= \int (3 \cos x - 3 \cos x \sin^2 x) \, dx \\ &= 3 \sin x - \sin^3 x + c\end{aligned}$$

$$\begin{aligned}(iii) \int 4 \tan x \, dx &= \int \frac{4 \sin x}{\cos x} \, dx \\ &= -4 \ln |\cos x| + c\end{aligned}$$

$$\begin{aligned}10. (i) \int_0^{\frac{\pi}{3}} 2 \sin^3 x \, dx &= \int_0^{\frac{\pi}{3}} 2 \sin x (1 - \cos^2 x) \, dx \\ &= \int_0^{\frac{\pi}{3}} (2 \sin x - 2 \sin x \cos^2 x) \, dx \\ &= \left[ -2 \cos x + \frac{2}{3} \cos^3 x \right]_0^{\pi/3} \\ &= -2 \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{8} - (-2 + \frac{2}{3}) \\ &= \frac{5}{12}\end{aligned}$$

$$\begin{aligned}(ii) \int_0^{\frac{\pi}{4}} \cos^2 x \, dx &= \int_0^{\frac{\pi}{4}} \frac{1}{2} (1 + \cos 2x) \, dx \\ &= \frac{1}{2} \left[ x + \frac{1}{2} \sin 2x \right]_0^{\pi/4} \\ &= \frac{1}{2} \times \frac{\pi}{4} + \frac{1}{4} - 0 \\ &= \frac{\pi}{8} + \frac{1}{4}\end{aligned}$$

$$\begin{aligned}(iii) \int_0^{\frac{\pi}{4}} \tan x \, dx &= \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x} \, dx \\ &= \left[ -\ln |\cos x| \right]_0^{\pi/4} \\ &= -\ln \left( \frac{1}{\sqrt{2}} \right) + \ln 1 \\ &= \ln(\sqrt{2}) = \frac{1}{2} \ln 2\end{aligned}$$