Section 2: Integration by substitution

Solutions to Exercise level 1

1. $u = 2x + 3 \Rightarrow \frac{du}{dx} = 2 \Rightarrow dx = \frac{1}{2}du$

$$\int (2x+3)^3 dx = \int u^3 \times \tfrac{1}{2} du$$
$$= \tfrac{1}{2} \times \tfrac{1}{4} u^4 + c$$
$$= \tfrac{1}{8}(2x+3)^4 + c$$

2. Let $u = 3 - 2x \Rightarrow \frac{du}{dx} = -2 \Rightarrow dx = -\frac{1}{2}du$

When x = -1, u = 5

When x = 1, u = 1

$$\int_{-1}^{1} (3-2x)^4 dx = \int_5^1 u^4 \times -\tfrac{1}{2} du$$
$$= \left[-\tfrac{1}{2} \times \tfrac{1}{5} u^5\right]_5^1$$
$$= -\tfrac{1}{10}\left(1^5 - 5^5\right)$$
$$= -\tfrac{1}{10} \times -3124$$
$$= 312.4$$

3. Let $u = 1 + 4x \Rightarrow \frac{du}{dx} = 4 \Rightarrow dx = \frac{1}{4}du$

When x = 0, u = 1

When x = 2, u = 9

$$\int_0^2 \sqrt{1+4x}\ dx = \int_1^9 u^{\frac{1}{2}} \times \tfrac{1}{4} du$$
$$= \left[\tfrac{1}{4} \times \tfrac{2}{3} u^{\frac{3}{2}}\right]_1^9$$
$$= \tfrac{1}{6}\left(9^{\frac{3}{2}} - 1^{\frac{3}{2}}\right)$$
$$= \tfrac{1}{6}(27 - 1)$$
$$= \tfrac{13}{3}$$

4. (i) $\int_1^2 \frac{1}{x} dx = [\ln|x|]_1^2$

$$= \ln 2 - \ln 1$$
$$= \ln 2$$

Edexcel A level Maths Integration 2 Exercise solutions

(ii) $\int_{-3}^{-1} \frac{1}{x} dx = [\ln|x|]_{-3}^{-1}$

$= \ln 1 - \ln 3$

$= -\ln 3$

5. (i) $\int_0^2 e^{3x} dx = \left[\tfrac{1}{3}e^{3x}\right]_0^2$

$= \tfrac{1}{3}\left(e^6 - 1\right)$

(ii) $\int_0^{\pi/4} \cos 2x \; dx = \left[\tfrac{1}{2}\sin 2x\right]_0^{\pi/4}$

$= \tfrac{1}{2}\left(\sin\tfrac{\pi}{2} - 0\right)$

$= \tfrac{1}{2}$

6. $\int_0^{\pi/4} (3\cos x - \sin x) \; dx = [3\sin x + \cos x]_0^{\pi/4}$

$= \left(3\sin\tfrac{\pi}{4} + \cos\tfrac{\pi}{4}\right) - \left(3\sin 0 + \cos 0\right)$

$= \tfrac{3}{\sqrt{2}} + \tfrac{1}{\sqrt{2}} - 1$

$= \tfrac{4}{\sqrt{2}} - 1$

$= 2\sqrt{2} - 1$

7. $u = x^3 + 2 \quad \Rightarrow \frac{du}{dx} = 3x^2 \quad \Rightarrow dx = \frac{1}{3x^2} du$

$\int x^2 \left(x^3 + 2\right)^3 dx = \int x^2 u^3 \times \frac{1}{3x^2} du$

$= \tfrac{1}{3}\int u^3 du$

$= \tfrac{1}{3} \times \tfrac{1}{4} u^4 + c$

$= \tfrac{1}{12}\left(x^3 + 2\right)^4 + c$

8. $u = x + x^2 \quad \Rightarrow \frac{du}{dx} = 1 + 2x \quad \Rightarrow dx = \frac{1}{1+2x} du$

$\int \frac{1+2x}{\sqrt{x+x^2}} dx = \int \frac{1+2x}{\sqrt{u}} \times \frac{1}{1+2x} du$

$= \int u^{-\frac{1}{2}} du$

$= 2u^{\frac{1}{2}} + c$

$= 2\sqrt{x + x^2} + c$

9\. (i) $$\int 2\sin^2 x\,dx = \int 2\times\tfrac{1}{2}(1-\cos 2x)\,dx$$
$$= \int (1-\cos 2x)\,dx$$
$$= x - \tfrac{1}{2}\sin 2x + c$$

(ii) $$\int 3\cos^3 x\,dx = \int 3\cos x(1-\sin^2 x)\,dx$$
$$= \int (3\cos x - 3\cos x\sin^2 x)\,dx$$
$$= 3\sin x - \sin^3 x + c$$

(iii) $$\int 4\tan x\,dx = \int \frac{4\sin x}{\cos x}\,dx$$
$$= -4\ln|\cos x| + c$$

10\. (i) $$\int_0^{\frac{\pi}{3}} 2\sin^3 x\,dx = \int_0^{\frac{\pi}{3}} 2\sin x(1-\cos^2 x)\,dx$$
$$= \int_0^{\frac{\pi}{3}} (2\sin x - 2\sin x\cos^2 x)\,dx$$
$$= \left[-2\cos x + \tfrac{2}{3}\cos^3 x\right]_0^{\pi/3}$$
$$= -2\times\tfrac{1}{2} + \tfrac{2}{3}\times\tfrac{1}{8} - \left(-2+\tfrac{2}{3}\right)$$
$$= \tfrac{5}{12}$$

(ii) $$\int_0^{\frac{\pi}{4}} \cos^2 x\,dx = \int_0^{\frac{\pi}{4}} \tfrac{1}{2}(1+\cos 2x)\,dx$$
$$= \tfrac{1}{2}\left[x + \tfrac{1}{2}\sin 2x\right]_0^{\pi/4}$$
$$= \tfrac{1}{2}\times\tfrac{\pi}{4} + \tfrac{1}{4} - 0$$
$$= \frac{\pi}{8} + \frac{1}{4}$$

(iii) $$\int_0^{\frac{\pi}{4}} \tan x\,dx = \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x}\,dx$$
$$= \left[-\ln|\cos x|\right]_0^{\pi/4}$$
$$= -\ln\left(\frac{1}{\sqrt{2}}\right) + \ln 1$$
$$= \ln\left(\sqrt{2}\right) = \tfrac{1}{2}\ln 2$$