

Section 1: Solving equations numerically

Solutions to Exercise level 1

1. Let $f(x) = e^x - 3x + 0.2$

$$f(1.34) = e^{1.34} - 3 \times 1.34 + 0.2 = -0.00096$$

$$f(1.35) = e^{1.35} - 3 \times 1.35 + 0.2 = 0.0074$$

Since there is a change of sign, there is a root between 1.34 and 1.35.

2. Let $f(x) = x^3 - 7x - 12$

$$f(3.2665) = 3.2665^3 - 7 \times 3.2665 - 12 = -0.0119$$

$$f(3.2675) = 3.2675^3 - 7 \times 3.2675 - 12 = 0.0131$$

Since there is a change of sign, there is a root between 3.2665 and 3.2675, so the root is 3.267 correct to 3 decimal places.

3. (i) $x^3 - x - 1 = 0$

$$x^3 = 1 + x$$

$$x = (1 + x)^{1/3}$$

(ii) $x_{n+1} = (1 + x_n)^{1/3}$

$$x_0 = 1$$

$$x_1 = (1 + x_0)^{1/3} = 2^{1/3} = 1.25992$$

$$x_2 = (1 + x_1)^{1/3} = 2.25992^{1/3} = 1.31229$$

$$x_3 = (1 + x_2)^{1/3} = 2.31229^{1/3} = 1.32235$$

$$x_4 = (1 + x_3)^{1/3} = 2.32235^{1/3} = 1.32427$$

$$x_5 = (1 + x_4)^{1/3} = 2.32427^{1/3} = 1.32463$$

$$x_6 = (1 + x_5)^{1/3} = 2.32463^{1/3} = 1.32470$$

(iii) The root is 1.325, correct to 3 d.p.

$$f(x) = x^3 - x - 1$$

$$f(1.3245) = -0.000929... < 0$$

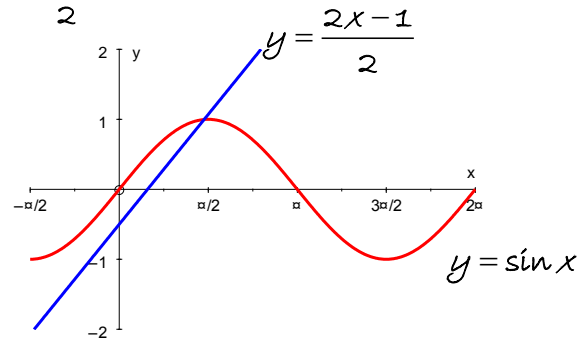
$$f(1.3255) = 0.00333... > 0$$

Since there is a change of sign in the interval (1.3245, 1.3255), the root must be 1.325 correct to 3 d.p.

Edexcel A level Maths Num methods 1 Exercise solns

4. (i) $2 \sin x - 2x + 1 = 0$

$$\sin x = \frac{2x-1}{2}$$



Since the graphs cross only once, the equation has only one real root.

(ii) $x_{n+1} = \sin x_n + 0.5$

Take $x_0 = 1$

$$x_1 = \sin x_0 + 0.5 = \sin 1 + 0.5 = 1.34147$$

$$x_2 = \sin x_1 + 0.5 = \sin 1.34147 + 0.5 = 1.47382$$

$$x_3 = \sin x_2 + 0.5 = \sin 1.47382 + 0.5 = 1.49530$$

$$x_4 = \sin x_3 + 0.5 = \sin 1.49530 + 0.5 = 1.49715$$

$$x_5 = \sin x_4 + 0.5 = \sin 1.49715 + 0.5 = 1.49729$$

The root is 1.497 (3 d.p.)

(iii) $f(x) = 2 \sin x - 2x + 1$

$$f(1.4965) = 0.00148... > 0$$

$$f(1.4975) = -0.000369... < 0$$

There is a change of sign in the interval (1.4965, 1.4975) so the root must be 1.497 correct to 3 d.p.