Section 1: The shape of curves

Solutions to Exercise level 2

1. (i) $y = x^4 - 2x^3$

$\frac{dy}{dx} = 4x^3 - 6x^2$

At stationary points, $4x^3 - 6x^2 = 0$

$x^2(2x-3) = 0$

$x = 0$ or $x = \frac{3}{2}$

When $x = 0$, $y = 0$

When $x = \frac{3}{2}$, $y = \left(\frac{3}{2}\right)^4 - 2\left(\frac{3}{2}\right)^3 = \frac{81}{16} - \frac{27}{4} = -\frac{27}{16}$

x	$x < 0$	$x = 0$	$0 < x < \frac{3}{2}$	$x = \frac{3}{2}$	$x > \frac{3}{2}$
$\frac{dy}{dx}$	-ve \	0 —	-ve \	0 —	+ve /

So (0, 0) is a point of inflection, and $\left(\frac{3}{2}, -\frac{27}{16}\right)$ is a minimum point.

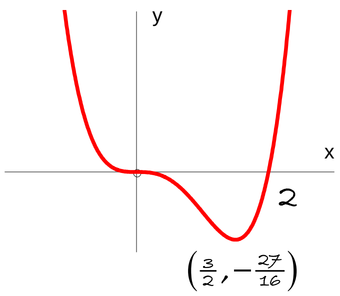
(ii) $\frac{d^2y}{dx^2} = 12x^2 - 12x = 12x(x-1)$

$\frac{d^2y}{dx^2} = 0$ at $x = 0$ (the stationary point of inflection) and at $x = 1$

When $x = 1$, $y = 1 - 2 = -1$

So (1, -1) is a non-stationary point of inflection

(iii)



2. (i) $y = 3x^4 - 16x^3 + 30x^2 - 24x + 12$

$$\Rightarrow \frac{dy}{dx} = 12x^3 - 48x^2 + 60x - 24$$

$$\Rightarrow \frac{d^2y}{dx^2} = 36x^2 - 96x + 60$$

(ii) $\frac{dy}{dx} = 0 \Rightarrow x^3 - 4x^2 + 5x - 2 = 0$

$$\Rightarrow (x-1)(x^2 - 3x + 2) = 0$$

$$\Rightarrow (x-1)(x-1)(x-2) = 0$$

$$\Rightarrow (x-1)^2(x-2) = 0$$

so the two points with zero gradient are (1, 5) (twice) and (2, 4).

(iii) Near (1, 5), checking e.g. $x = 0.9 \Rightarrow \frac{dy}{dx} \approx -0.1$

$x = 1.1 \Rightarrow \frac{dy}{dx} \approx -0.1$

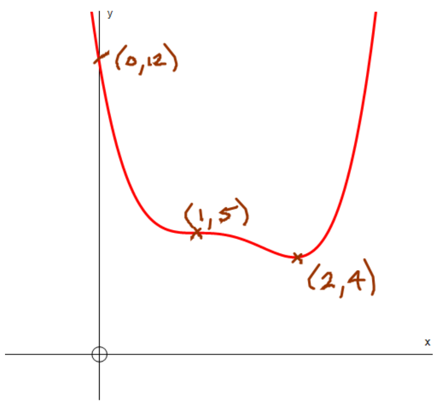
so (1, 5) is a point of inflection.

Near (2, 4), checking e.g. $x = 1.9 \Rightarrow \frac{dy}{dx} \approx -0.97$

$x = 2.1 \Rightarrow \frac{dy}{dx} \approx +1.5$

so (2, 4) is a local minimum.

(iv) The graph crosses the y-axis at (0, 12).



Edexcel A level Maths Differentiation 1 Exercise solns

3. (i) $y = (x+1)(x-3)^3$

$= (x+1)(x^3 - 9x^2 + 27x - 27)$

$= x^4 - 9x^3 + 27x^2 - 27x + x^3 - 9x^2 + 27x - 27$

$= x^4 - 8x^3 + 18x^2 - 27$

(ii) At turning points, $\frac{dy}{dx} = 0$

$4x^3 - 24x^2 + 36x = 0$

$x^3 - 6x^2 + 9x = 0$

$x(x^2 - 6x + 9) = 0$

$x(x-3)^2 = 0$

$x = 0$ or $x = 3$

When $x = 0$, $y = (0+1)(0-3)^3 = 1 \times -27 = -27$

When $x = 3$, $y = 0$

The turning points are (0, -27) and (3, 0).

(iii) $\frac{d^2y}{dx^2} = 12x^2 - 48x + 36$

When $x = 0$, $\frac{d^2y}{dx^2} = 36 > 0$, so (0, -27) is a minimum.

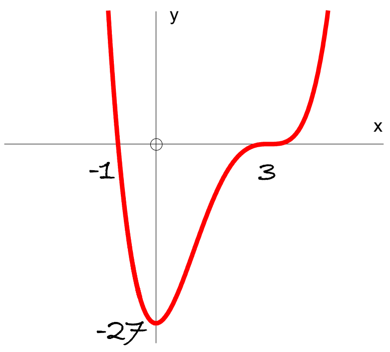
When $x = 3$, $\frac{d^2y}{dx^2} = 108 - 144 + 36 = 0$

To left of t.p: when $x = 2$, $\frac{dy}{dx} = 32 - 96 + 72 > 0$

To right of t.p: when $x = 4$, $\frac{dy}{dx} = 256 - 96 + 144 > 0$

so (3, 0) is a point of inflection.

(iv) When $y = 0$, $x = -1$ or $x = 3$



4. (i) $y = \frac{1}{x} - x^2 + 3x$

$\frac{dy}{dx} = -x^{-2} - 2x + 3$

At stationary points, $-x^{-2} - 2x + 3 = 0$

$-1 - 2x^3 + 3x^2 = 0$

$2x^3 - 3x^2 + 1 = 0$

By inspection $x = 1$ is a root, so $(x-1)$ is a factor

$(x-1)(2x^2 - x - 1) = 0$

$(x-1)(2x+1)(x-1) = 0$

Stationary points are at $x = -\frac{1}{2}$ and $x = 1$

When $x = -\frac{1}{2}$, $y = -2 - \frac{1}{4} - \frac{3}{2} = -\frac{15}{4}$

When $x = 1$, $y = 1 - 1 + 3 = 3$

$\frac{d^2y}{dx^2} = 2x^{-3} - 2$

At $x = -\frac{1}{2}$, $\frac{d^2y}{dx^2} < 0$ so $\left(-\frac{1}{2}, -\frac{15}{4}\right)$ is a local maximum

At $x = 1$, $\frac{d^2y}{dx^2} = 0$

To the left of $x = 1$, $\frac{dy}{dx} < 0$ and to the right of $x = 1$, $\frac{dy}{dx} < 0$

so (1, 3) is a point of inflection.

5. Since this is a cubic curve with a stationary point of inflection, it must be a transformation of the curve $y = x^3$. Since the coefficient of x^3 is 1, it cannot involve a stretch, so it must be a translation. The origin has been translated to (-1, 3) so the equation of the curve is $y = (x+1)^3 + 3$

$= x^3 + 3x^2 + 3x + 1 + 3$

$= x^3 + 3x^2 + 3x + 4$

Alternatively you could approach this problem by differentiating twice and setting both the first and second derivatives to zero at $x = -1$