

## Section 1: Introduction to differential equations

## Solutions to Exercise level 2

$$1. \quad x \frac{dy}{dx} = \frac{1}{\cos y}$$

$$\int \cos y dy = \int \frac{1}{x} dx$$

$$\sin y = \ln|x| + c$$

$$\text{When } x=1, y = \frac{3}{4}\pi \Rightarrow \sin \frac{3}{4}\pi = \ln 1 + c \Rightarrow c = \frac{1}{\sqrt{2}}$$

$$\sin y = \ln|x| + \frac{1}{\sqrt{2}}$$

$$2. \quad \frac{dy}{dx} = -xy$$

$$\int \frac{1}{y} dx = \int -x dx$$

$$\ln|y| = -\frac{1}{2}x^2 + c$$

$$\text{When } x=0, y=1 \Rightarrow \ln 1 = 0 + c \Rightarrow c=0$$

$$\ln|y| = -\frac{1}{2}x^2$$

$$y = e^{-\frac{1}{2}x^2}$$

$$3. \quad x + y \frac{dy}{dx} = 10$$

$$y \frac{dy}{dx} = 10 - x$$

$$\int y dy = \int (10 - x) dx$$

$$\frac{1}{2}y^2 = 10x - \frac{1}{2}x^2 + c$$

$$y^2 = 20x - x^2 + k$$

$$4. \quad \frac{dm}{dt} = -\frac{m}{(1+t)^2}$$

$$\int \frac{1}{m} dm = \int -(1+t)^{-2} dt$$

$$\ln m = (1+t)^{-1} + c$$

$$\text{When } t=0, m=10 \Rightarrow \ln 10 = 1 + c \Rightarrow c = \ln 10 - 1$$

## Edexcel A level Maths Diff eqns 1 Exercise solns

$$\ln m = \frac{1}{1+t} + \ln 10 - 1$$

$$\ln m - \ln 10 = \frac{1}{1+t} - 1 = \frac{1 - (1+t)}{1+t}$$

$$\ln \frac{m}{10} = -\frac{t}{1+t}$$

$$\frac{m}{10} = e^{-\frac{t}{1+t}}$$

$$m = 10e^{-\frac{t}{1+t}}$$

When  $t$  becomes very large,  $\frac{t}{1+t}$  is very close to 1.

$$\text{So } m \approx 10e^{-1} \approx 3.7$$

So there will be about 3.7 kg left.

5. 
$$\frac{dy}{dx} = \frac{2y}{x^2 - 1}$$

$$\int \frac{1}{y} dy = \int \frac{2}{(x+1)(x-1)} dx$$

using partial fractions, 
$$\frac{2}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$
$$2 = A(x-1) + B(x+1)$$

Putting  $x=1 \Rightarrow 2 = 2B \Rightarrow B=1$

Putting  $x=-1 \Rightarrow 2 = -2A \Rightarrow A=-1$

$$\int \frac{1}{y} dy = \int \left( \frac{1}{x-1} - \frac{1}{x+1} \right) dx$$

$$\ln|y| = \ln|x-1| - \ln|x+1| + c$$

$$\ln|y| = \ln \left| \frac{A(x-1)}{x+1} \right|$$

$$y = A \left( \frac{x-1}{x+1} \right)$$

replacing  $c$  by  $\ln A$