# Section 3: Implicit differentiation

## Solutions to Exercise level 3

1. $x^2 + xy + 2y^2 = 8$

Differentiating: $2x + x\dfrac{dy}{dx} + y + 4y\dfrac{dy}{dx} = 0$

Gradient of normal at (a, b) is 4, so gradient of tangent is $-\frac{1}{4}$

Substituting $x = a, y = b$ and $\dfrac{dy}{dx} = -\frac{1}{4}$

gives $2a - \frac{1}{4}a + b + 4b \times -\frac{1}{4} = 0$

$2a - \frac{1}{4}a + b - b = 0$

$2a - \frac{1}{4}a = 0$

$a = 0$

Substituting $x = 0$ and $y = b$ into $x^2 + xy + 2y^2 = 8$

gives $2b^2 = 8$

$b^2 = 4$

$b = \pm 2$

So (a, b) is (0, 2) or (0, -2).

2. (i) $y = xe^x$

$\dfrac{dy}{dx} = xe^x + e^x$

When $x = -1$, gradient $= -e^{-1} + e^{-1} = 0$

(ii) $\ln y = \ln(xe^x)$

$= \ln x + \ln e^x$

$= \ln x + x$

Differentiating: $\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{x} + 1$

When $x = -1$, $\dfrac{1}{y}\dfrac{dy}{dx} = -1 + 1 = 0$ so gradient $= 0$

3. (i) $y = \tan x = \dfrac{\sin x}{\cos x}$

$$\frac{dy}{dx} = \frac{\cos x \times \cos x - \sin x \times -\sin x}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= 1 + \tan^2 x$$

(ii) $\tan x + \tan y = 4$

Differentiating: $(1 + \tan^2 x) + (1 + \tan^2 y)\dfrac{dy}{dx} = 0$

When $x = \dfrac{\pi}{4}$, $\tan x = 1$ and therefore $\tan y = 3$

$$(1 + 1^2) + (1 + 3^2)\frac{dy}{dx} = 0$$

$$2 + 10\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{1}{5}$$